Bishop's University

Math 314: Test
Winter 2014

- This test is 80 minutes in length.
- All answers are to be given to three decimal places unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- Total Points: 77

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1. (7 points) It is claimed that a new diet will reduce a person's weight by 4.5kg on average in a period of 2 weeks. The weights of 7 women who followed this diet were recorded before and after the two week period.

Woman	Before	After
1	58.5	60.0
2	60.3	54.9
3	61.7	58.1
4	69.0	62.1
5	64.0	58.5
6	62.6	59.9
7	56.7	54.4

Test the claim about the diet by computing a 95% confidence interval for the mean difference in weights. You may assume the difference in weights are approximately normally distributed.

2. A survey is to be done with the hopes of comparing salaries of chemical plant managers employed in two different areas of the country.

   (a) (5 points) An independent sample of 300 plant managers was selected for each of the two regions, and the results were as follows:

	North	South
mean	\$102,300	\$98,500
standard deviation	\$5,700	\$3,800

   Based on this data, construct a 99% confidence interval for $\mu_1 - \mu_2$. You may assume that the population variances are equal.

   (b) (4 points) What is the minimum sample size (use the same sample size for both areas) is necessary to produce a 95% confidence interval on $\mu_1 - \mu_2$ having width of only \$1000? Previous statistics suggest that $\sigma_1 = \sigma_2 = \$4000$.

3. (5 points) A random sample of 30 firms dealing in wireless products was selected to determine the proportion of such firms that have implemented new software to improve productivity. It turns out that 8 of the 30 had implemented such software. Find a 95% confidence interval on p, the true proportion of such firms that have implemented new software.

4. (3 points) How large a sample is required if the power of the test is to be 0.80 when the true mean exceeds the hypothesized value by 0.4σ? Use $\alpha = 0.025$.

5. (5 points) The contents of containers of a particular lubricant is known to be normally distributed with a variance of 0.03. Test the hypothesis that $\sigma^2 = 0.03$ against the alternative that $\sigma^2 \neq 0.03$, for a sample of 10 containers with sample variance of 0.06. Use a 0.01 level of significance.

6. (7 points) A random sample of 90 adults is classified according to gender and the number of hours of television watched during a week:

	Male	Female
Over 25 hours	15	29
Under 25 hours	27	19

Use a 0.01 level of significance and test the hypothesis that the time spent watching television is independent of the gender of the viewer.

7. (7 points) A coin is tossed until a head occurs and the number X of tosses recorded. After repeating the experiment 256 times, we obtain the following results:

x	1	2	3	4	5	6	7	8
f	136	60	34	12	9	1	3	1

Test the hypothesis at the 0.05 level of significance that the observed distribution of X may be fitted by the geometric distribution $g(x; 1/2) = (1/2)^x$, $x = 1, 2, 3 \dots$.

8. (7 points) A geneticist is interested in the proportions of males and females in a population who have a certain minor blood disorder. If a random sample of 100 males, 31 are found to be affected, whereas only 24 of 100 females tested have the disorder. Can we conclude at the 0.01 level of significance that the proportion of men in the population afflicted with this blood disorder is significantly greater than the proportion of women afflicted?

9. (7 points) If one can containing 500 nuts is selected at random from each of three different distributors of mixed nuts and there are, respectively 345, 313, and 359 peanuts in each of the cans, can we conclude at the 0.05 level of significance that the mixed nuts of the three distributors contain equal proportions of peanuts?

10. (6 points) Consider a Poisson distribution with probability mass function

$$f(x|\mu) = \frac{e^{-\mu}\mu^x}{x!}, \quad x = 0, 1, 2, \dots$$

Suppose that a random sample $x_1, x_2, \dots, x_n$ is taken from the distribution. What is the maximum likelihood estimate of μ?

11. (3 points) A group of researchers are concerned about reaction to stimulus by airplane pilots. An experiment was conducted and 40 pilots were used with an average reaction time of 3.7 seconds with a sample standard deviation of 0.7 seconds. It is of interest to characterize the worst case scenario. Compute a one-sided tolerance bound with 95% confidence that involves 95% of reaction times. Give an interpretation of the result. You may assume that the measurements come from a normal distribution.

12. In Math 196, data concerning the submission of assignments and final grades were collected, and is as follows:

Missed assignments	0	4	3	5	2	0	2	0
Final Grade	51	65	69	59	33	80	58	63

(a) (7 points) Find the equation of the regression line which estimates the final grade based on the number of assignments missed.

(b) (2 points) Estimate the final grade of a student who missed 1 assignment. The answer must be a whole number.

13. (1 point) What is "Type II" error?

14. (1 point) What is a prediction interval used for?