Bishop's University

Math 314: Final Exam
Winter 2016

- This test is 180 minutes in length.
- All answers are to be given to three decimal places unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- Total Points: 90

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1. (10 points) It is claimed that a new diet will reduce a person's weight by 4.0kg on average in a period of 2 weeks. The weights of 7 women who followed this diet were recorded before and after the two week period.

Woman	Before	After
1	58.5	54.9
2	60.3	60.0
3	61.7	62.1
4	69.0	58.1
5	64.0	59.9
6	62.6	58.5
7	56.7	54.4

Test the claim about the diet by computing a 95% confidence interval for the mean difference in weights. You may assume the difference in weights are approximately normally distributed. The variance in the difference of weights is 13.726.

2. A survey is to be done with the hopes of comparing salaries of chemical plant managers employed in two different areas of the country.
   (a) (5 points) An independent sample of 300 plant managers was selected for each of the two regions, and the results were as follows:

	North	South
mean	\$103,200	\$97,500
standard deviation	\$6,700	\$3,500

   Based on this data, construct a 99% confidence interval for $\mu_1 - \mu_2$. You may assume that the population variances are equal.

   (b) (5 points) What is the minimum sample size (use the same sample size for both areas) is necessary to produce a 95% confidence interval on $\mu_1 - \mu_2$ having width of only \$1000? Previous statistics suggest that $\sigma_1 = \sigma_2 = \$4000$.

3. (5 points) A random sample of 30 firms dealing in wireless products was selected to determine the proportion of such firms that have implemented new software to improve productivity. It turns out that 7 of the 30 had implemented such software. Find a 95% confidence interval on p, the true proportion of such firms that have implemented new software.

4. (5 points) How large a sample is required if the power of the test is to be 0.90 when the true mean exceeds the hypothesized value by 0.5σ? Use $\alpha = 0.01$.

5. (5 points) The contents of containers of a particular lubricant is known to be normally distributed with a variance of 0.04. Test the hypothesis that $\sigma^2 = 0.04$ against the alternative that $\sigma^2 \neq 0.04$, for a sample of 10 containers with sample variance of 0.09. Use a 0.01 level of significance.

6. (10 points) A random sample of 90 adults is classified according to gender and the number of hours of television watched during a week:

	Male	Female
Over 25 hours	29	19
Under 25 hours	27	15

Use a 0.02 level of significance and test the hypothesis that the time spent watching television is independent of the gender of the viewer.

7. (10 points) A coin is tossed until a head occurs and the number X of tosses recorded. After repeating the experiment 256 times, we obtain the following results:

x	1	2	3	4	5	6	7	8
f	123	60	37	17	12	1	3	3

Test the hypothesis at the 0.05 level of significance that the observed distribution of X may be fitted by the geometric distribution $g(x; 1/2) = (1/2)^x$, $x = 1, 2, 3 \dots$.

8. (5 points) A geneticist is interested in the proportions of males and females in a population who have a certain minor blood disorder. If a random sample of 100 males, 34 are found to be affected, whereas only 20 of 100 females tested have the disorder. Can we conclude at the 0.01 level of significance that the proportion of men in the population afflicted with this blood disorder is significantly greater than the proportion of women afflicted?

9. (5 points) If one can containing 500 nuts is selected at random from each of three different distributors of mixed nuts and there are, respectively 315, 373, and 339 peanuts in each of the cans, can we conclude at the 0.05 level of significance that the mixed nuts of the three distributors contain equal proportions of peanuts?

10. In Math 196, data concerning the submission of assignments and final grades from 8 students was collected, and is as follows:

Missed assignments	0	4	3	1	0	2	1	9
Final Grade	68	45	39	45	73	51	72	59

(a) (7 points) Find the equation of the fitted line which estimates the final grade based on the number of assignments missed.

(b) (5 points) Find the coefficient of determination for the fitted model.

(c) (4 points) Find a 95% confidence interval for the true mean response when 1 assignment is missed.

(d) (4 points) If a student who missed 5 assignments earned a final grade of 65, can you be 90% confident that this is an outlier?

11. (5 points) If we have regression data $\{(x_i, y_i) | i = 1, \dots, n\}$ the fitted linear model is $\widehat{y} = b_0 + b_1 x$, prove that the regression data $\{(y_i, x_i) | i = 1, \dots, n\}$ has a fitted model $\widehat{x} = a_0 + a_1 y$ where $a_0 = -\dfrac{b_0}{b_1}$ and $a_1 = \dfrac{1}{b_1}$.