Bishop's University

Math 315: Final Exam
Winter 2023

Name: ______________________________

Student #: ______________________________

- This test is 180 minutes in length.
- **There are 100 possible points available on this test. The results will be graded out of 90 marks to a maximum of 90/90. To say it differently, there are 10 bonus marks on this test.**
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- You are permitted to use one (1) **Authorized Memory Book** and a **Casio fx-260 Solar (II) calculator**.
- Do not remove any pages from this test.
- All answers must be written in the space provided.
- The back of each page may be used for scrap paper.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	10	
3	10	
4	10	
5	10	
6	20	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points) Use the definition of convergence to prove the following sequence converges: $\left\{ \frac{2-2n}{n} \right\}_{n=1}^{\infty}$.

2. (10 points) Prove directly (without assuming convergent sequences are Cauchy) that if $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are Cauchy, so is $\{a_n b_n\}_{n=1}^{\infty}$. You may use the fact that Cauchy sequences are bounded.

3. (10 points) Define $f:(0,1)\to\mathbb{R}$ by $f(x)=\dfrac{x^3-x^2+x-1}{x-1}$. Use the definition to prove that f has a limit at $x=1$.

4. (10 points) A function $f : \mathbb{R} \to \mathbb{R}$ is periodic if and only if there is a real number $h > 0$ such that $f(x+h) = f(x)$ for all $x \in \mathbb{R}$. Prove that if $f : \mathbb{R} \to \mathbb{R}$ is periodic and continuous, then f is uniformly continuous.

5. (10 points) Let $E_1, \ldots, E_n$ be compact. Prove that $\cup_{i=1}^n E_i$ is compact.

6. (10 points) Find an interval of length 1 that contains a root of the equation $x^3 - 6x^2 + 2.826 = 0$.

7. (10 points) Prove that if $f : A \to \mathbb{R}$ is monotone and $1-1$, then f^{-1} is monotone and $1-1$.

8. (10 points) Determine if the series $\sum_{m=1}^{\infty} \frac{\sqrt{m+1}-\sqrt{m}}{m}$ converges or diverges.

9. (10 points) Test the series $\sum_{n=1}^{\infty} n^p p^n$, $p > 0$ for convergence.

10. (10 points) Suppose the series $\sum_{n=0}^{\infty} a_n$ converges conditionally. Find all values $x \in \mathbb{R}$ such that $\sum_{n=0}^{\infty} a_n x^n$ converges.