Bishop's University

Math 317: Final Exam
Fall 2014

- This test is 180 minutes in length.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Unless directed otherwise, express your answers in the form $x + iy$.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- In the exam booklet, clearly identify the question you are answering.

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1. (5 points) Write $(2-2i)^7$ in complex exponential form with an argument in the interval $(5\pi, 7\pi]$.
2. (5 points) Write $7e^{-i\frac{27}{6}\pi}$ in rectangluar form. All trigonometric functions must be evaluated.
3. (5 points) Express in the form $a + bi$, $a, b \in \mathbb{R}$, all solutions of $\cosh z = -3i$.
4. (4 points) Solve $\text{Re}\,(\sinh z) = 0$.
5. (4 points) For what values of z is it true that $\text{Ln}\,\frac{1}{z} = -\text{Ln}\,z$.
6. (4 points) Give an example where it is not true that $\text{Arg}\,(zw) = \text{Arg}\,z + \text{Arg}\,w$. The numbers z and w must be in rectangular form, fully evaluated, with no trigonometric functions remaining.
7. (a) (2 points) State the Cauchy-Riemann equations.
   (b) (4 points) Without reference to analyticity, verify that the Cauchy-Riemann equations are satisfied for the function $f(z) = z^3$.
   (c) (3 points) Show that if the function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, show that the function
   $$g(z) = -5\left(\overline{f(2i - \overline{z})}\right)$$
   is also analytic in D.
   (d) (3 points) If $u(x, y) = ax^3 + bx^2y + cxy^2 + dy^3$, what are (necessary) conditions on the constants $a, b, c, d \in \mathbb{C}$ to guarantee that u is harmonic?
8. (4 points) Classify the singular points of $\dfrac{e^{\frac{1}{z}}}{(z-i)^2(z+3-2i)^3}$.
9. (8 points) Find all Laurent series for
   $$f(z) = \frac{z+1}{z^2+2z+2}$$
   about $z = -1$ and state where they are convergent. Hint: complete the square in the denominator.
10. (5 points) If $f(z) = \displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n!}(z+3i)^{n+2}$, evaluate $\displaystyle\oint_C \frac{f(z)}{(z+3i)^5}$ where C is $|z| = 4$, travelled counterclockwise.
11. (5 points) Compute $\displaystyle\oint_C \frac{1}{z \sin z}dz$ where C is the curve $|z| = 1$ travelled counterclockwise.
12. (6 points) Find the principal value of $\displaystyle\int_{-\infty}^{\infty} \frac{(x+1)^2}{x^4+16}dx$
13. Consider the linear fractional transformation that sends
    $$-1 \text{ to } 1$$
    $$i \text{ to } 0$$
    $$1 \text{ to } -1.$$
    (a) (4 points) Find the transformation.
    (b) (3 points) What does the interior of $|z| = 1$ get mapped to? You may sketch your answer.
    (c) (3 points) What are the fixed points?