Bishop's University

Math 317: Final Exam
Fall 2016

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

- All answers must be exact (no decimals allowed) unless specifically directed otherwise.

- A **Casio fx260-solar** calculator is permitted. No other electronic calculators are permitted.

1. (5 points) Write $(-1+i\sqrt{3})^7$ in complex exponential form with an argument in the interval $(-3\pi, -\pi]$.

2. (5 points) Write $5e^{i\frac{23}{6}\pi}$ in rectangluar form. All trigonometric functions must be evaluated.

3. (5 points) Express in the form $a + bi$, $a, b \in \mathbb{R}$, all solutions of $z^3 + 8i = 0$. All trigonometric functions must be evaluated.

4. (5 points) Solve: $\text{Im}(\cosh z) = 0$.

5. (a) (2 points) State the Cauchy-Riemann equations.

   (b) (4 points) Without reference to analyticity, verify that the Cauchy-Riemann equations are satisfied for the function $f(z) = z^3$.

   (c) (3 points) If f is a non-constant real-valued function, show that f is not analytic.

   (d) (6 points) If $u(x, y) = ax^2 + bxy + cy^2$ is harmonic, find its harmonic conjugate. Are there any restrictions on a, b, and c?

6. (8 points) Classify the singular points of $\dfrac{\cos\left(\frac{1}{z}\right)}{(z^2 + 1)^2(z + 3)^3}$.

7. (10 points) Find all Laurent series for

$$f(z) = \frac{z - 2}{z^2 - 4z + 5}$$

   about $z = 2$ and state where they are convergent. **Hint: complete the square in the denominator.**

8. (5 points) If $f(z) = \sum_{n=0}^{\infty} \dfrac{(-1)^{n+1}2^n}{n!(n+1)!}(z - 3)^{n+2}$, evaluate $\oint_C \dfrac{f(z)}{(z-3)^7}$ where C is $|z| = 4$, travelled counterclockwise.

9. (5 points) Compute $\oint_C \dfrac{1}{z \sinh z} dz$ where C is the curve $|z| = 1$ travelled counterclockwise.

10. (6 points) Find the principal value of $\displaystyle\int_{-\infty}^{\infty} \dfrac{x^2}{x^4 + 1} dx$

11. Consider a linear fractional transformation that sends

$$5 \text{ to } -2 + i$$
$$10 - 2i \text{ to } 2 + i$$
$$2i \text{ to } -i.$$

   (a) (6 points) Find the transformation.

   (b) (6 points) What does the upper half-plane $2x + 5y > 10$ get mapped to?

   (c) (4 points) What are the fixed points?