BISHOP'S UNIVERSITY

MATH 317: FINAL EXAM
FALL 2018

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

- All answers must be exact (no decimals allowed) unless specifically directed otherwise.

- All answers must be in the form of $x + iy$ unless otherwise indicated.

- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.

- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

1. (5 points) Write $(\sqrt{3} - i)^5$ in complex exponential form with an argument in the interval $(\pi, 3\pi]$.

2. (5 points) Write $4e^{-i\frac{17}{6}\pi}$ in rectangular form. All trigonometric functions must be evaluated exactly.

3. (5 points) Express in the form $a + bi$, $a, b \in \mathbb{R}$, all solutions of $z^4 + 5z^2 = 36$. All trigonometric functions must be evaluated.

4. (5 points) Solve: $\sin z = \cosh 3$.

5. (a) (2 points) State the Cauchy-Riemann equations.

   (b) (6 points) Without reference to analyticity, verify that the Cauchy-Riemann equations are satisfied for the function $f(z) = \dfrac{1}{z^2}$.

   (c) (6 points) If $u(x, y) = ax + by$ is harmonic, find its harmonic conjugate. Are there any restrictions on a and b?

6. (10 points) Classify the singular points of $\dfrac{\sin z}{z^5(z^2 + 1)^2(z + 3)^3}$.

7. (10 points) Find all Laurent series for
$$f(z) = \frac{3z}{8z^3 - 1}$$
about $z = 0$ and state where they are convergent.

8. (5 points) If $f(z) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}2^n}{3n(n+1)!}(z + i)^{5-n}$, evaluate $\displaystyle\oint_C \frac{f(z)}{(z + i)^2}$ where C is $|z - i| = 4$, travelled counterclockwise.

9. (5 points) Compute $\displaystyle\oint_C \frac{e^z}{\cos \pi z}dz$ where C is the curve $|z| = 1$ travelled counterclockwise.

10. (10 points) Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 - 4\sin\theta}$

11. Consider a linear fractional transformation that maps

$$\infty \text{ to } 1$$
$$0 \text{ to } 0$$
$$-1 \text{ to } \frac{1+i}{2}.$$

   (a) (6 points) Find the transformation.

   (b) (6 points) What does the upper half-plane $y > 0$ get mapped to?

   (c) (4 points) What are the fixed points?