Bishop's University

Math 317: Final Exam
Fall 2020

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- All answers must be in the form of $x + iy$ unless otherwise indicated.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	10	
3	10	
4	10	
5	15	
6	10	
7	10	
8	10	
9	15	
Total:	90	

1. (5 points) Write $(-1-i)^3$ in complex exponential form with an argument in the interval $(-5\pi, -3\pi]$.

2. (5 points) Write $6e^{i\frac{41}{3}\pi}$ in rectangular form. All trigonometric functions must be evaluated exactly.

3. (5 points) Express in the form $a+bi$, $a, b \in \mathbb{R}$, all solutions of $z^6+7z^3 = 8$. All trigonometric functions must be evaluated.

4. (5 points) Solve: $\sin z = 3$.

5. (10 points) Let $u(x, y) = ax^2 + bxy + cy^2$. Find any restrictions on a, b and c necessary so that u is harmonic, and find its harmonic conjugate.

6. (10 points) Classify the singular points of $\dfrac{\sin^2 z}{z^5(z^2+9)^3(z+3\pi)^2}$.

7. (5 points) Let $f(z) = \dfrac{p(z)}{q(z)}$, where p, q are polynomials with degree of p greater than the degree of q. Prove that f has a singularity at ∞,

8. (10 points) Find all Laurent series for

$$f(z) = \frac{7z^2}{z^2 - 9}$$

about $z = 0$ and state where they are convergent.

9. (5 points) Evaluate $\oint_C z^5 \cos\left(\frac{1}{z}\right) dz$ where C is $|z - i| = 4$, travelled counterclockwise.

10. (5 points) Compute $\oint_C \frac{e^z}{\cos \pi z} dz$ where C is the curve $|z| = 2$ travelled counterclockwise.

11. (10 points) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(4+x^2)^2}$

12. Consider a linear fractional transformation that maps

$$\begin{aligned} \infty &\text{ to } \frac{1+i}{2} \\ 0 &\text{ to } 1+i \\ i &\text{ to } 1. \end{aligned}$$

(a) (5 points) Find the transformation.

(b) (5 points) What does the left half-plane $x < 0$ get mapped to?

(c) (5 points) What are the fixed points?