BISHOP'S UNIVERSITY

MATH 421/CS 471/CS 569: FINAL EXAM
SPRING 2019

Name: _____

Student #: _____


- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

- All answers must be exact (no decimals allowed) unless specifically directed otherwise.

- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.

- This exam is 3 hours in length.

- Do not remove any pages from this test.

- The back of each page may be used for scrap paper.

Page	Points	Score
2	6	
3	15	
4	12	
5	10	
6	15	
7	15	
8	10	
Total:	83	

1. For the graph below, answer the following questions. Justification must be given in order to get full marks.
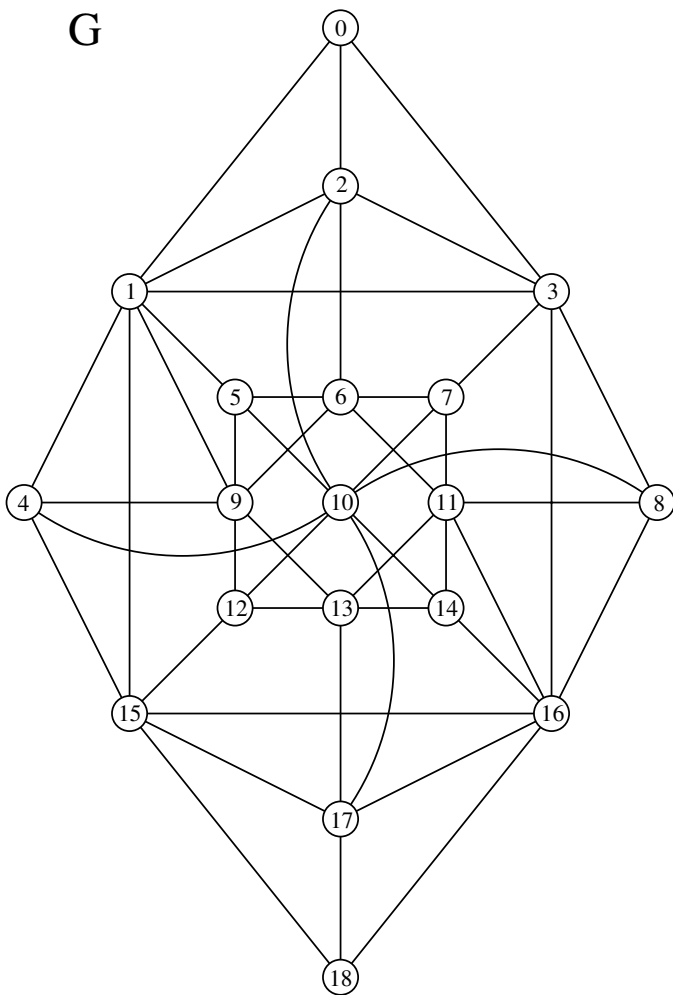
(a) (3 points) Is the graph Eulerian? If so, give the Eulerian circuit.

(b) (3 points) Is the graph Hamiltonian? If so, give the Hamiltonian cycle.

(c) (3 points) Is the graph bipartite?

(d) (3 points) Write the degree sequence of the graph.

(e) (3 points) How many edges in the graph?

(f) (3 points) What is the chromatic number of the graph?

(g) (3 points) To solve the Chinese Postman problem, which edges need to be added, if any. (You do not need to compute all the possible permutations to answer this question)

(h) (6 points) Use a depth-first search, starting at 0, to find a spanning tree for this graph. Label the vertices and give the final backtracking.
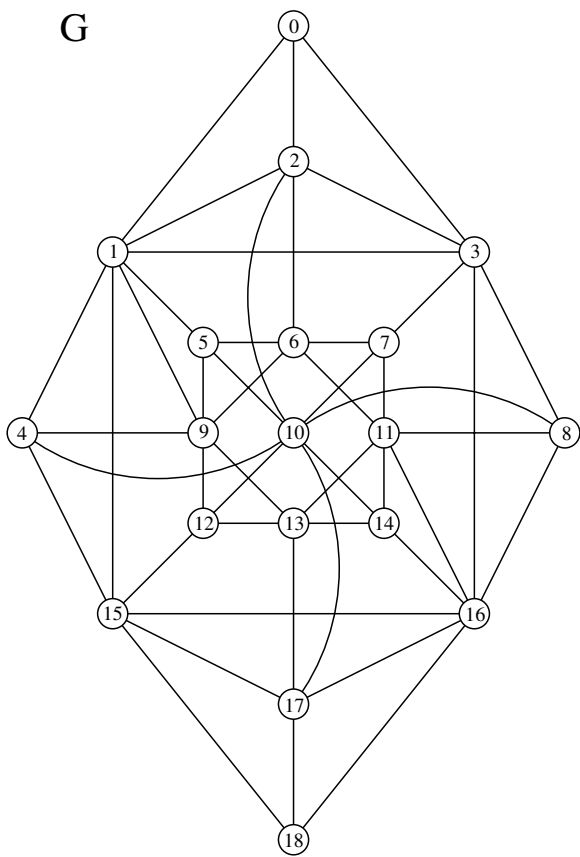
(i) (6 points) Orient the graph so that it is strongly connected. (Hint: use the result from (h).)
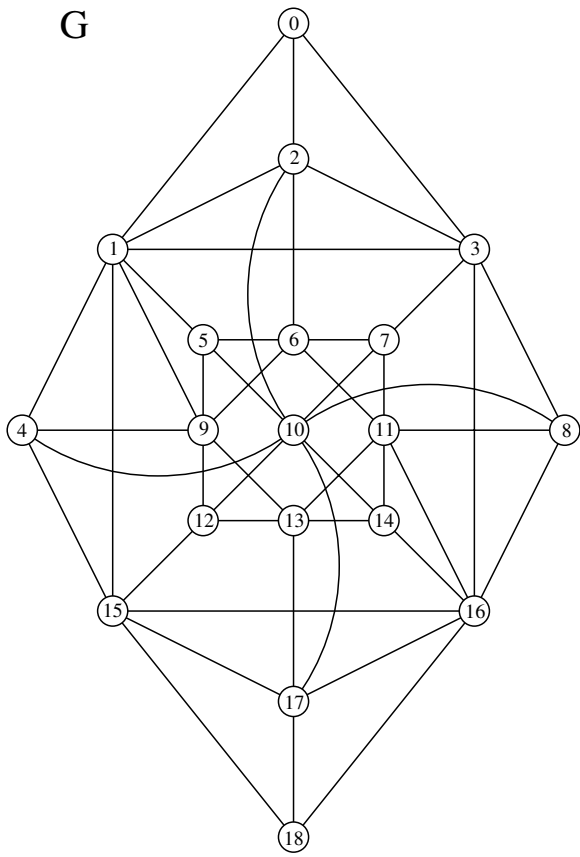
2. (10 points) Suppose we want to construct a widget. The times for the various steps are given as follows:

	Activity	Time (in minutes)
F_1	Construct foundation	5
F_2	Attach five flanges	4
A_1	Add antenna	8
B_1	Secure bolt in place	15
A_2	Slide in axles	3
W	Affix all three wheels	2
B_2	Remove bolt	1
C	Insert clamps	4
P	Two coats of paint	2

The foundation has to be constructed before anything else can be done. The antenna has to be added and the bolt secured before the axles are slid in or the wheels are affixes. The axles must be in and the wheels affixed before the bolt is removed. The flanges must be attached before the bolt is secured. The clamps are inserted after the axles are in. The widget is painted after the wheels are affixed. The widget is completed after the clamps are inserted, the bolt is removed and the widget is painted.

Use the provided (unlabeled) graph to find the least amount of time required for this project and give the critical path or paths. (You need to label the graph for this scheduling problem.)
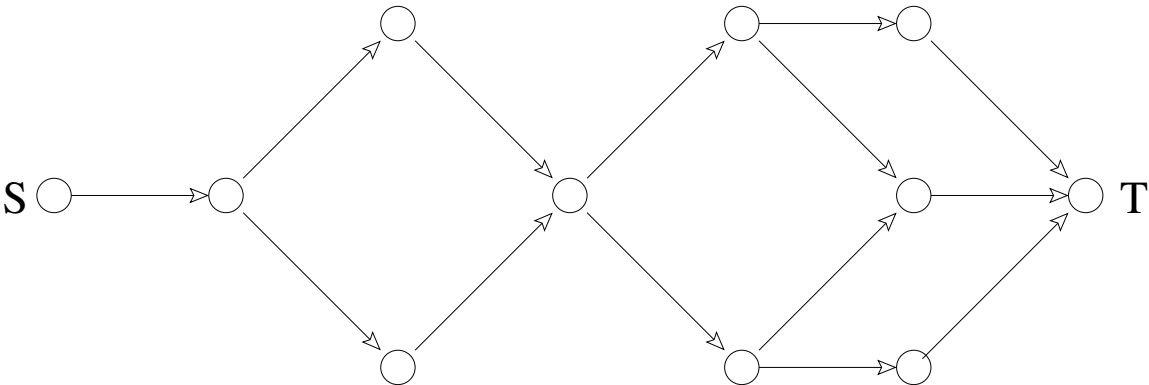
3. (15 points) Given the following weighted graph, find the shortest distance from u to v using Dijkstra's algorithm (either the original or the improved algorithm). Find a minimum spanning tree for the same graph.
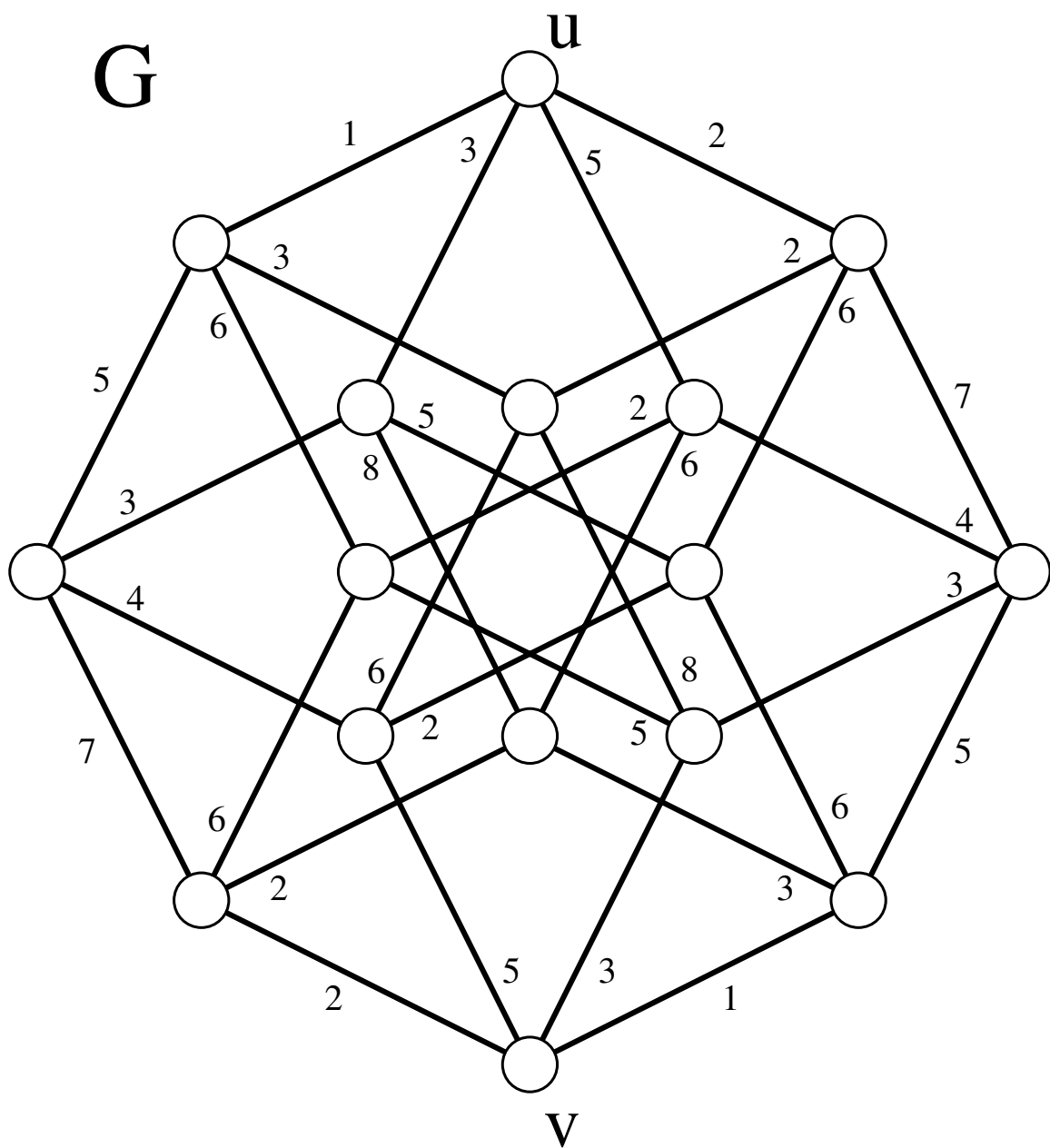
4. (15 points) For the following network, with source s and sink t, find a maximal flow and a minimal cut.
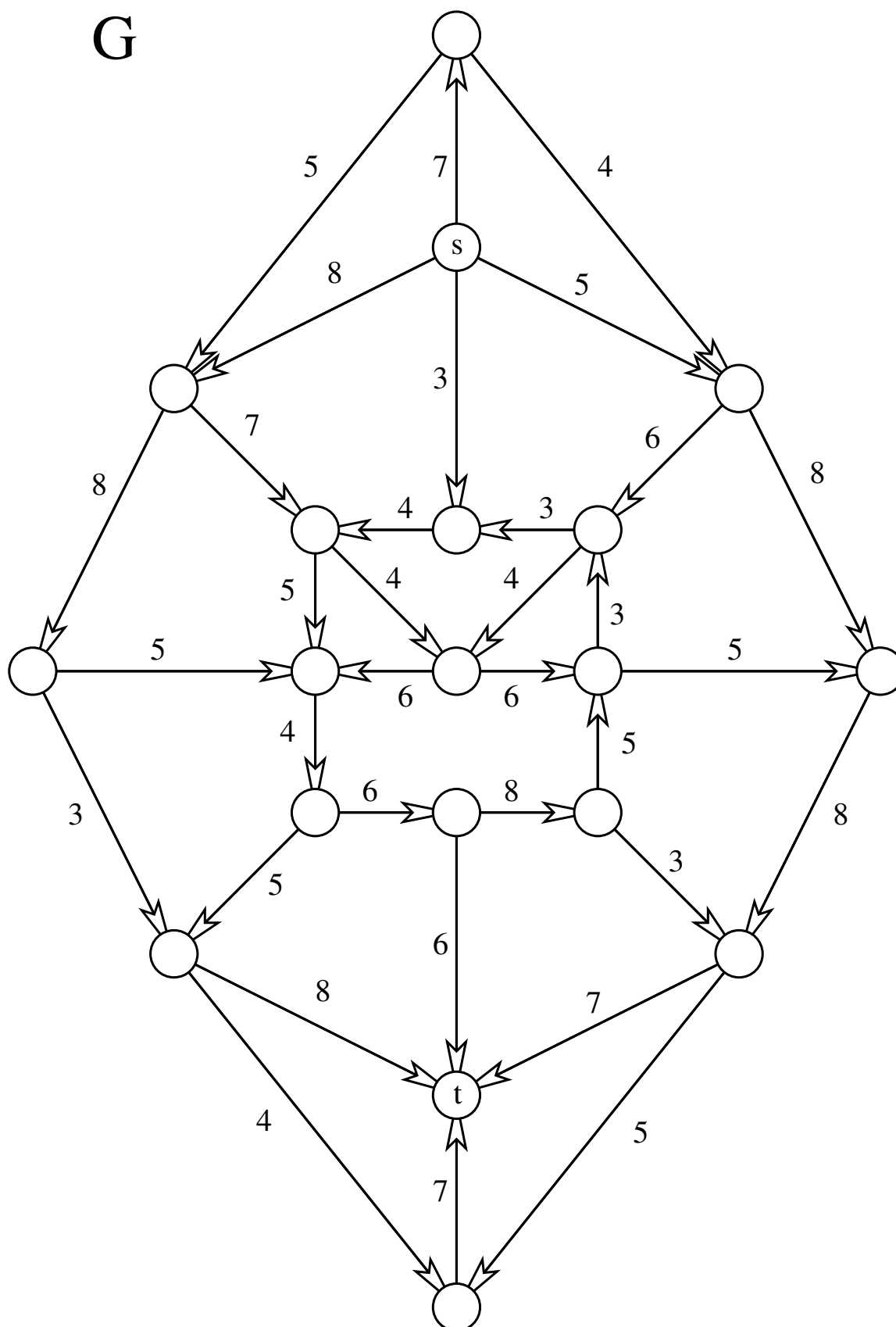
5. (10 points) If a connected graph, G contains a bridge, e, prove that any spanning tree for G must contain the edge e.