# Bishop's University

## Math 421/CS 471/CS 571: Final Exam
## Fall 2022

**Name:** _____

**Student #:** _____

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- This test is 180 minutes in length.
- All answers must be written in the space provided.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- You are permitted to use one (1) Authorized Memory Book and a Casio fx-260 Solar (II) calculator.

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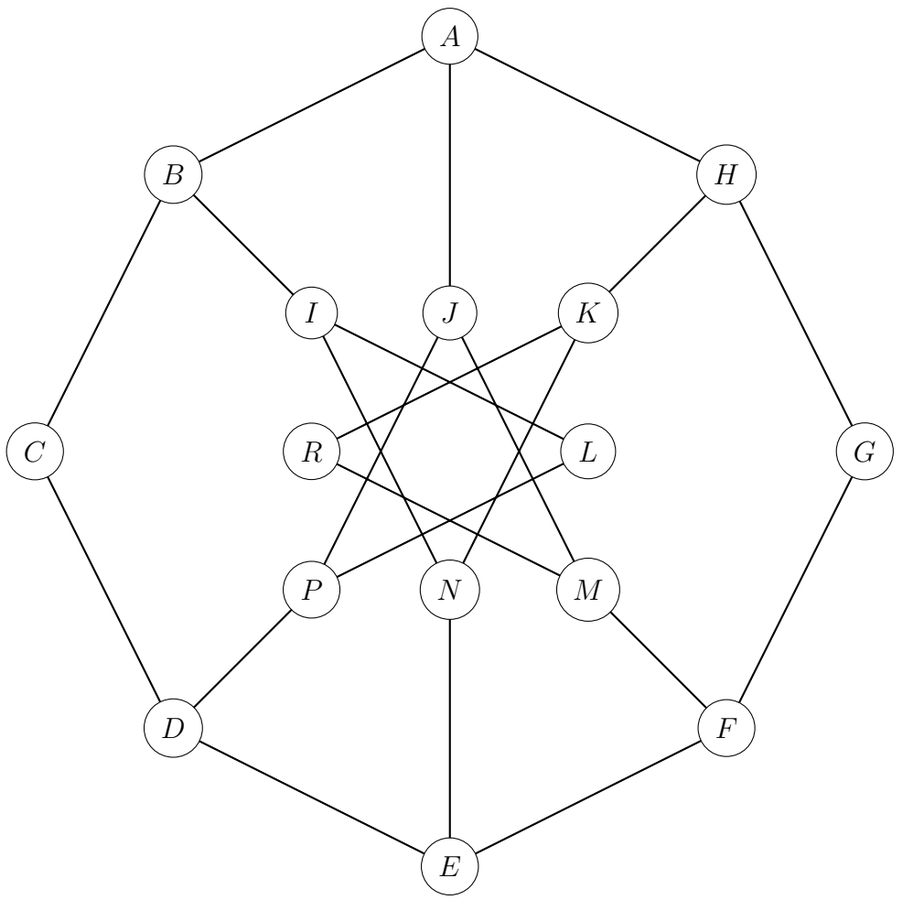
| Page | Points | Score |
|---|---|---|
| 2 | 10 | |
| 3 | 25 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 15 | |
| 7 | 10 | |
| 8 | 15 | |
| 9 | 5 | |
| Total: | 100 | |

1. For the graph below, answer the following questions. This graph is used on the next page as well. **Justification must be given in order to get full marks.**

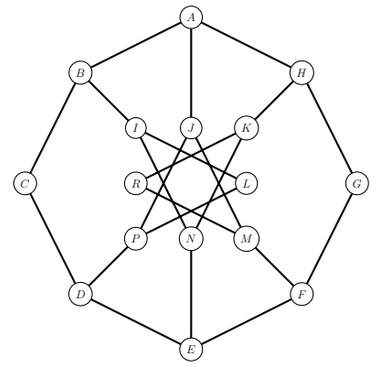



(a) (5 points) Is the graph Eulerian? If so, give the Eulerian circuit.

(b) (5 points) Is the graph Hamiltonian? If so, give the Hamiltonian cycle starting at vertex $A$ and includes the edge $AJ$.

(c) (5 points) Is the graph bipartite?

Reference graph for this page



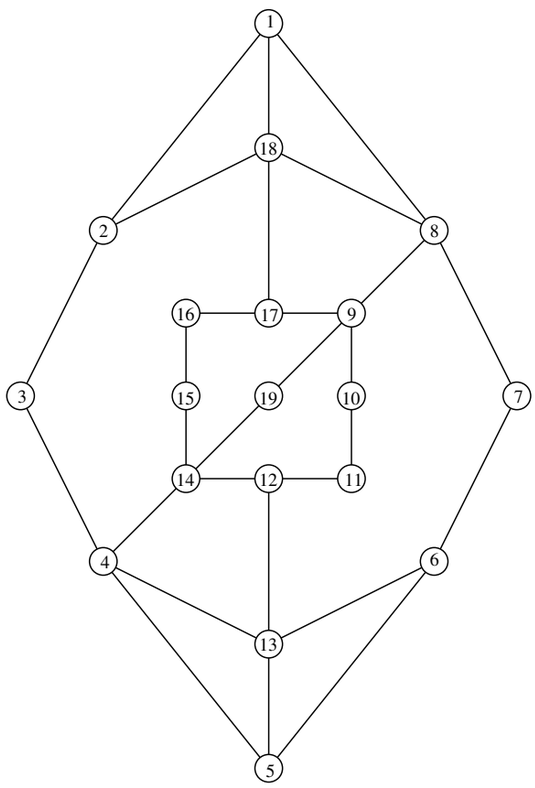

(d) (5 points) Write the degree sequence of the graph.

(e) (5 points) How many edges in the graph?

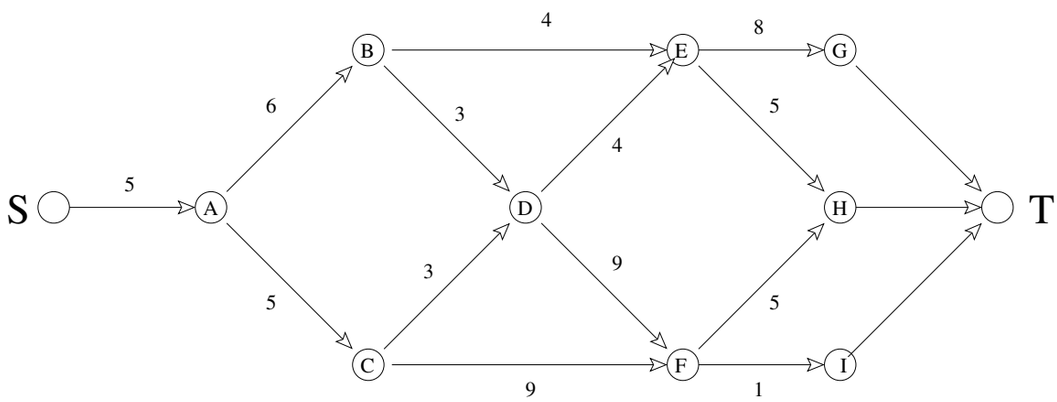
(f) (5 points) What is the chromatic number of the graph?

(g) (5 points) To solve the Chinese Postman problem, which edges need to be added, if any. (You do not need to compute all the possible permutations to answer this question)

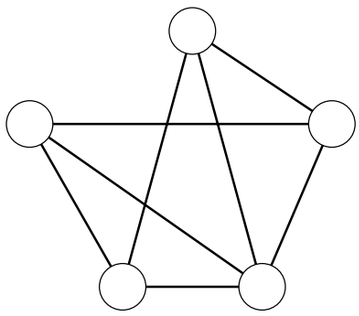
2. (10 points) The graph is labelled using a depth-first search. Use this labelling to orient the graph so that it is strongly connected. Indicate on the graph the spanning tree and give the final backtracking. You may draw the orientation directly on the provided graph.



3. (10 points) Consider the following network as representing a Type II scheduling problem. Find a critical path from $S$ to $T$ and give the overall time required to complete such a project. Give the slack for each vertex not on the critical path.



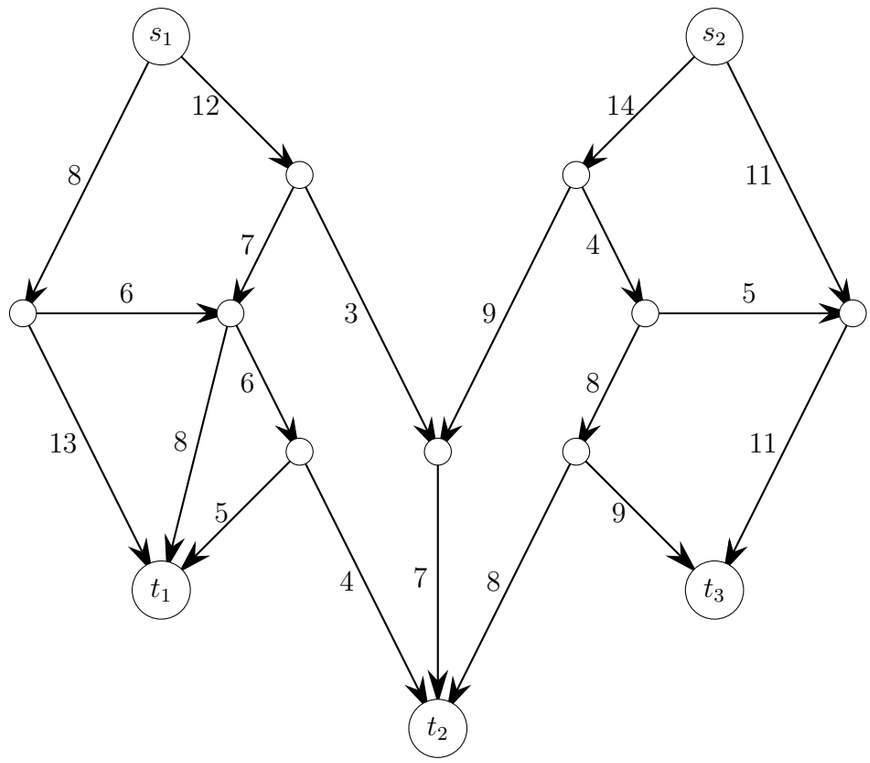
4. (15 points) Find the number of spanning trees for the following graph.



5. (10 points) Below is the distance matrix $d$ at step $k = 2$ from the Floyd-Warshall Algorithm for shortest distances between any two vertices. Give the distance matrix $d$ for the next step ($k = 3$).

$$d = \begin{bmatrix} 0 & 6 & 9 & 7 & 4 \\ 6 & 0 & 3 & 13 & 10 \\ 9 & 3 & 0 & 2 & 13 \\ 7 & 13 & 2 & 0 & 1 \\ 4 & 10 & 13 & 1 & 0 \end{bmatrix}$$

6. (15 points) For the following network, with sources $s_1$ and $s_2$ and sinks $t_1, t_2$, and $t_3$, find a maximal flow and a minimal cut. Label the vertices on the graph as necessary.



7. (5 points) Prove that a connected graph $G$ is bipartite if and only if the chromatic number of $G$ is 2.