Fractional Arithmetic

1. **How to multiply two fractions:** To indicate multiplication we use one of three methods: juxtapostion, the symbol $\cdot$, or the symbol $\times$. They are related as follows: $ab = a \cdot b = a \times b$. Brackets may be used for clarification. So to multiply two fractions, we have

$$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{a}{b} \cdot \frac{c}{d} = \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

   For example, $\dfrac{3}{7} \cdot \dfrac{5}{4} = \dfrac{15}{28}$.

2. **How to reduce fractions:** If we can write the fraction $\dfrac{a}{b}$ in the form $\dfrac{mk}{nk}$, then we can say that $\dfrac{a}{b} = \dfrac{m}{n}$. The reason this is true is as follows:

$$\frac{a}{b} = \frac{mk}{nk} = \frac{m}{n} \cdot \frac{k}{k} = \frac{m}{n} \cdot 1 = \frac{m}{n}.$$

   For example, $\dfrac{120}{72} = \dfrac{5}{3}$, since $120 = 5 \cdot 24$ and $72 = 3 \cdot 24$.

3. **How to divide two fractions:** There are two ways to indicate division, one is as a fraction, the other is with the division symbol $\div$. They are related as follows: $a \div b = \dfrac{a}{b}$. Now, to divide two fractions, we have the rule

$$\frac{a}{b} \div \frac{c}{d} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{a}{b} \cdot \frac{d}{c}$$

   which reduces division to the process of multiplication. For example, $\dfrac{3}{7} \div \dfrac{5}{4} = \dfrac{12}{35}$.

4. **How to add (or subtract) two fractions:** To add two fractions we use the following rule:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}.$$

   This rule arises from adding fractions with a common denominator, in that

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

   when we reduce the fraction. To see how this applies to the above situation, consider

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} + \frac{b}{b} \cdot \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd}.$$

   As an example of addition, $\dfrac{2}{3} + \dfrac{3}{4} = \dfrac{17}{12}$.