Exponentials and Logarithms

Let a be any positive real number not equal to 1. Then we have two functions, the exponential a^x, and the logarithm $\log_a x$, which are related in the following manner:

$$a^x = y \qquad \text{is equivalent to} \qquad \log_a y = x$$

1. **Properties of Exponents:** Let a and b be positive real numbers.
   (a) $a^{x+y} = a^x a^y$. For example, $2^{3+5} = 2^3 \cdot 2^5$.
   (b) $a^{x-y} = \dfrac{a^x}{a^y}$. For example, $7^{11-8} = \dfrac{7^{11}}{7^8}$.
   (c) $(a^x)^y = a^{xy}$. For example, $(4^3)^5 = 4^{15}$.
   (d) $(ab)^x = a^x b^x$. For example, $(5 \cdot 3)^2 = 5^2 \cdot 3^2$.
   (e) $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$. For example, $\left(\dfrac{4}{9}\right)^3 = \dfrac{4^3}{9^3}$.
   (f) $a^0 = 1$. For example, $6^0 = 1$.
   (g) $a^1 = a$. For example, $17^1 = 17$.
   (h) $1^x = 1$. For example, $1^{\sqrt{2056395}} = 1$.
   (i) $a^{-x} = \dfrac{1}{a^x}$. For example, $5^{-4} = \dfrac{1}{5^4}$.
   (j) $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$, where m and n are positive integers.
   For example, $8^{\frac{7}{3}} = \left(\sqrt[3]{8}\right)^7 = \sqrt[3]{8^7}$.

2. **Properties of Logarithms:** Let a and b be positive real numbers not equal to 1, let x and y be positive real numbers, and let r be any real number.
   (a) $\log_a xy = \log_a x + \log_a y$. For example, $\log_2(3 \cdot 5) = \log_2 3 + \log_2 5$.
   (b) $\log_a \dfrac{x}{y} = \log_a x - \log_a y$. For example, $\log_7 \dfrac{3}{8} = \log_7 3 - \log_7 8$.
   (c) $\log_a x^r = r \log_a x$. For example, $\log_5 10^4 = 4 \log_5 10$.
   (d) $\log_a 1 = 0$. For example, $\log_{13} 1 = 0$.
   (e) $\log_a a = 1$. For example, $\log_{11} 11 = 1$.
   (f) $\log_a \dfrac{1}{x} = -\log_a x$. For example, $\log_9 \dfrac{1}{27} = -\log_9 27$.
   (g) $\log_a x = \dfrac{\log_b x}{\log_b a}$. For example, $\log_3 12 = \dfrac{\log_2 12}{\log_2 3}$.
   (h) $\log x = \log_{10} x$.
   (i) $\ln x = \log_e x$.

3. **Cancellation Properties of Exponentials and Logarithms:** Let a be any positive real number not equal to 1.
   (a) $\log_a a^x = x$ for any real number x. For example, $\log_6 6^{-12} = -12$.
   (b) $a^{\log_a x} = x$ for any positive real number x. For example, $8^{\log_8 \frac{4}{5}} = \dfrac{4}{5}$.