Note: You must show your work in order to receive full marks

FINAL EXAM   **JUNE 26, 2007**

<u>MARKS</u>

1. Find the derivative of each of the following functions (DO NOT SIMPLIFY):

(4)   (a) $y = \dfrac{1+x}{1+x^2} - x^{\frac{1}{3}} + 5$ [**ans.**]

(4)   (b) $y = x^3 3^x$ [**ans.**]

(4)   (c) $y = \ln(1 + e^x)$ [**ans.**]

(4)   (d) $y = x^{\ln x}$ [**ans.**]

(4)   (e) $y = (\sinh x)^2$ [**ans.**]
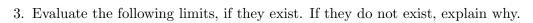
(4) (f) $y = \arctan(\cos x)$ [ans.]

(4) (g) $y = \ln\left(\dfrac{x^2 - 1}{x}\right)$ [ans.]

2. For the curve $x^3 + xy + y^3 = 11$

(4) (a) Find y'. [ans.]

(4) (b) Find the equation of the tangent line at $(1, 2)$. [ans.]

(3) (c) Find y''. [ans.]

3. Evaluate the following limits, if they exist. If they do not exist, explain why.

(4)   (a) $\lim\limits_{x\to 2} \dfrac{x^2 + 4}{x + 2}$ [ans.]

(4)   (b) $\lim\limits_{x\to 0} \dfrac{\arctan x}{x}$ [ans.]

(4)   (c) $\lim\limits_{x\to\infty} \dfrac{\sinh x}{\cosh x}$ [ans.]

(4)   (d) $\lim\limits_{x\to 0^+} x \ln x$ [ans.]

(4)   (e) $\lim\limits_{x\to 0} \dfrac{|x|}{x}$ [ans.]

(4)   (f) $\displaystyle\lim_{x \to -2^-} \frac{1}{4 - x^2}$ [ans.]

(4)   (g) $\displaystyle\lim_{x \to 0} \frac{1 - \cos x}{x^2}$ [ans.]

(6)   4. Use the logarithmic derivative to find $f'(x)$ [ans.]

$$f(x) = \frac{\sqrt[4]{x}(x + 1)^5}{(5 - 3x)^{\frac{2}{7}}} e^{\tanh x}$$

(4)   5. Find $f(\ln 5)$ if $f'(x) = \cosh x - e^{2x}$ and $f(0) = 3$.   [ans.]

(10)   6. For the function $y = f(x) = x^3 - 3x^2$ determine: [ans.]

      (a) intervals of increase and decrease, all local maxima and minima;

      (b) intervals of concavity and inflection points;

      (c) sketch the graph of $y = f(x)$.

(10)   7. A box with a square base and open top must have a volume of $12 ft^3$. Materials for the bottom cost $\$3/ft^2$ while material for the sides cost $\$1/ft^2$. Find the dimensions of the box which minimizes the total cost. [ans.]

      8. The volume V of a spherical balloon is increasing at a rate of $3cm^3/sec$. (Hints: $V = 4\pi r^3/3$, $S = 4\pi r^2$)

(4)   (a) Find the rate of change of the radius r when $r = 5cm$ [ans.]

(2)   (b) Find the rate of change of the surface area S when $r = 5cm$ [ans.]