Math 1003: Final Exam

Student Number: _____

No electronic devices are allowed. For full marks, all work must be shown.
This exam is three (3) hours in duration. Total marks possible: 105/100

1. Evaluate the following limits. If the limit is infinite, state whether it is postive or negative infinity. Justify your answers.

   (a) (3 points) $\lim_{h \to 2} \dfrac{h^3 - 8}{h^2 - 4}$

   (b) (3 points) $\lim_{x \to -3^-} \dfrac{x + 9}{x^2 - 9}$

   (c) (3 points) $\lim_{x \to 0} \dfrac{\tan x}{x \cos x}$

   (d) (3 points) $\lim_{x \to 1^-} \dfrac{\arctan x}{\arcsin x}$

   (e) (3 points) $\lim_{x \to \infty} \dfrac{\ln x}{x}$

2. Differentiate the following functions. DO NOT SIMPLIFY!

(a) (4 points) $y = x^3 \cos(\sqrt{x} + 5)$

(b) (4 points) $y = \tan(\sin(\pi x))$

(c) (4 points) $y = e^{\cosh^2 x}$

(d) (4 points) $y = \dfrac{\sec\theta - \ln 9}{\tan\theta}$

(e) (4 points) $y = \ln(\cot^4 x \csc^4 x)$ **(Hint: simplify the logarithm first)**

(f) (4 points) $y = \dfrac{1}{\sqrt[3]{x^4 + 5x^2 + 10}}$

(g) (4 points) $y = (x^2 + 4)^{5x}$

3. (a) (7 points) Use the **definition of the derivative** to find $f'(x)$ for $f(x) = \sqrt{x+5}$.

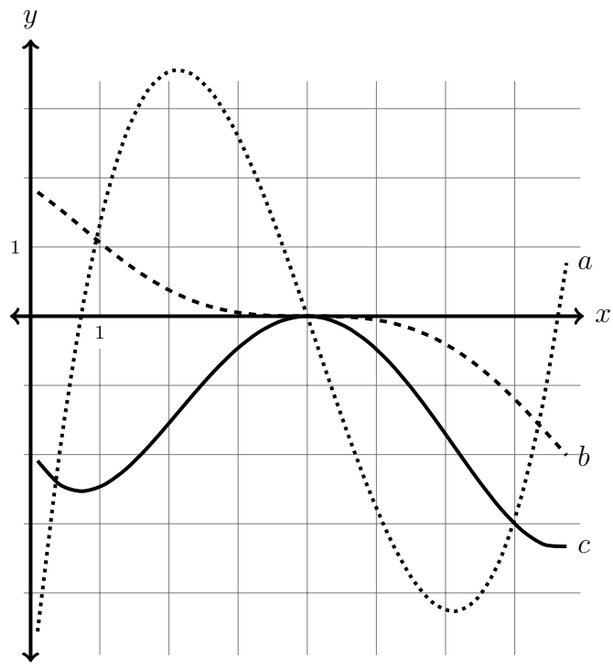
(b) (3 points) Using the information from part (a), write the equation of the tangent line to $y = f(x)$ at $x = 4$.

4. (7 points) Show that the graph of $x^2 - 3xy + y^2 = 1$ has no horizontal tangent lines.

5. (5 points) The limit below represents a derivative $f'(a)$. Identify $f(x)$ and a. What is the value of the limit?

$$\lim_{h \to 0} \frac{5^{2+h} - 25}{h}$$

6. (3 points) The figure below shows the graphs of the functions f, f', and f''. Label each function.



7. (3 points) Find the absolute maximum value of $f(x) = 9x^{1/3} - 6\sqrt{x}$ on the interval $[0, 64]$.

8. (8 points) The included angle of the two sides of constant equal length s of an isosceles triangle is θ. Show that the area of the the triangle is $A = \frac{1}{2}s^2 \sin\theta$. If θ is increasing at a rate of $\frac{1}{2}$ radians per minute, find the rate of change of the area when $\theta = \pi/6$.
(Hint: the double-angle formula $\sin 2x = 2\sin x \cos x$ **may be useful.)**

9. (8 points) A poster is to have a total area of 180cm^2, and a margin of 1cm at the bottom and the sides and a margin of 2cm at the top. What dimensions give the largest useable area?

10. (12 points) Sketch the graph of $y = \dfrac{x^3 - 3x + 3}{x - 1}$.

The grid is on the next page, so there is plenty of room to show your work!

Use the first and second derivatives to analyze the shape. Consider (1) domain, (2) local maxima and minima, (3) intervals of increase and decrease, (4) concavity, (5) inflection points, and (6) asymptotes, in your analysis. Label the important features on your graph. The first and second derivatives are given below:

$$y' = \frac{x^2(2x - 3)}{(x - 1)^2} \qquad y'' = \frac{2x(x^2 - 3x + 3)}{(x - 1)^3}$$

11. (6 points) For the following questions, all answers are to be written as a single (reduced) fraction or integer.

$\frac{10}{5} + \frac{10}{6} =$	$\frac{6}{8} + \frac{3}{12} =$	$\frac{4}{9} + \frac{9}{2} =$
$\frac{4}{11} - \frac{1}{6} =$	$\frac{1}{4} - \frac{2}{12} =$	$\frac{8}{6} - \frac{11}{3} =$