Math 1013: Final Exam

Name:____ **ID #:**_________________________

Question 1 . . . . *28 points*

Evaluate the following integrals:

(a) (4 points) $\displaystyle\int x^2 - 2^x \, dx$

(b) (4 points) $\displaystyle\int \sec x \, dx$

(c) (4 points) $\displaystyle\int_0^9 \frac{\sin\left(\pi\sqrt{25-x}\right)}{\sqrt{25-x}} \, dx$

(d) (4 points) $\displaystyle\int \tan^3\theta \sec\theta \, d\theta$

(e) (4 points) $\int t^2 e^{-2t}\, dt$

(f) (4 points) $\int_0^3 \frac{1}{\sqrt{y^2+9}}\, dy$

(g) (4 points) $\int \frac{y^3+y+1}{y^2(y^2+1)}\, dy$

Question 2 .... *8 points*

Determine whether the following improper integrals converge or diverge. If the integral converges, find the value to which it converges.

(a) (4 points) $\displaystyle\int_1^\infty e^{-x}\,dx$

(b) (4 points) $\displaystyle\int_{-1}^{8} \frac{1}{\sqrt[3]{x}}\,dx$

Question 3 .... *16 points*

Solve the following differential equations:

(a) (4 points) $y' = e^{x+y}$

(b) (4 points) $xy' + 2y = x^2 + 1$

(c) (4 points) $y'' + 4y' + 3y = 0$, with $y(0) = 0$ and $y'(0) = 2$

(d) (4 points) $y'' + 9y = 0$, with $y(0) = 1$ and $y(\pi/4) = 1$

Question 4 .... *4 points*

Find the area bound by the curves $y = x^3 - 3x^2 + 2x$ and $y = 3x - 3$. (Hint: the point $(1, 0)$ is common to both curves.)

Question 5 .... *4 points*

Find the volume of the solid generated by rotating the region bound by $x = y$ and $x = y^2$ about the x-axis.

Question 6 .... *4 points*

Use the Talyor polynomial of degree 2 for $f(x) = \sqrt{x}$, centered at $x = 100$ to approximate $\sqrt{99}$.

Question 7 .... *6 points*

Use the definition of the definite integral as the limit of a Riemann sum to compute $\int_0^4 3x + 1 \; dx$.

Question 8 . . . . *4 points*

Let $g(x) = \displaystyle\int_x^{x^2} \frac{1+t}{1-t}\, dt$. Is g continuous at $x = 1$? Justify your response.

Question 9 . . . . *4 points*

Find the average value of $y = \sin^2(nx)$ on the interval $[0, \pi/n]$, where n is a positive integer.

Question 10 . . . . *4 points*

The speedometer of a car travelling north on a straight road was observed at 1 minute intervals as indicated here:

time: t (minutes)	0	1	2	3	4	5	6
velocity: v (km/h)	0	50	60	70	100	100	0

Use Simpson's Rule to estimate the distance (in kilometers) the car travelled during these 6 minutes.

Question 11 .... *18 points*

Choose three of the following six problems. Clearly identify which three questions you want graded. Extra questions will not be graded. Use the three pages at the end of this exam for these questions. Each of the three questions will be worth 6 points.

(a) A piece of rope is to be used to lift a bucket from the ground to the top of a 10 meter building. The bucket and its content weighs 10 kilograms. The rope weighs 1 kg per meter. How much work is done lifting the bucket from the ground to the top of the building? (You may assume that $g = 10m/s^2$.)

(b) A tank contains 1000 litres of brine, consisting of 10 kg salt and water. Pure water enters the tank at a rate of 5 L/min. The well mixed solution drains from the tank, also at 5 L/min. At what time does the tank contain exactly 5 kg of salt?

(c) The rate of growth of a bacterial population P(t) is known to be proportional to P(t) itself. Set up and solve a differential equation which describes this population. If there are initially 2000 bacteria, then 4000 after 2 hours, about how many bacteria were there after 1 hour? (You may leave your answer in terms of well-known constants.)

(d) Find the orthogonal trajectories of the family of curves given by $kx^2 + y^2 = 1$.

(e) Find the length of the curve given by $x = \dfrac{y^3}{6} + \dfrac{1}{2y}$ with $2 \leq y \leq 3$.

(f) A solid is built on the circle $x^2 + y^2 = 4$ as base. Each cross-section perpendicular to the x-axis is a square. Find its volume.