UNIVERSITY OF NEW BRUNSWICK, SAINT JOHN
DEPARTMENT OF MATHEMATICAL SCIENCES
SPRING 2006

MATH 1503
INTRODUCTION TO LINEAR ALGEBRA

Final Exam

Name: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Student Number: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Time: 3 hours

Mark: ⎯⎯⎯⎯⎯/50 (Maximum mark: 52)

Instructions:

• Show all your work in this booklet.

• Work neatly and in an organized manner.

• If you run out of space in a problem, use the space on the back of the page
  and clearly indicate where the solution continues.

• Do not remove any pages from this booklet.

• Good luck!

1. Let
$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}.$$

(a) (3 marks) Find the eigenvalues of A. (Hint: $\lambda = -2$ is an eigenvalue.)

(b) (3 marks) Find the eigenvectors of A.

(c) (5 marks) Diagonalize A.

(d) (2 mark) Use the information above to find A^4.

2. (4 marks) Find the equation of the plane containing the points $(1, 0, 1)$, $(0, -1, -1)$, and $(-1, 1, 0)$.

3. (5 marks) Let
$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

which reduces to

$$B = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find a basis for the row space, column space, and null space of A. Also compute the rank and nullity of A.

4. (3 marks) Is

$$S = \left\{ \begin{bmatrix} x \\ xy \\ y \end{bmatrix} \middle| x, y \in \mathbb{R} \right\}$$

a subspace of $\mathbb{R}^3$? Give reasons.

5. (3 marks) Find the distance between the point $(1, -2, 3)$ and the plane $3x - y + z = -1$.

6. (2 marks) Find the cosecant of the angle between the planes $x - y + z = 1$ and $3x - y + z = -1$.

7. (3 marks) Are the following vectors linearly independent? Give reasons.

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

8. (3 marks) Solve, if possible:

$$\begin{array}{rcl} 2r + s & = & 3 \\ 4r + s & = & 7 \\ 2r + 5s & = & -1 \end{array}$$

9. (3 marks) Describe the span of the following vectors:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \\ 18 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

10. Let $z = 1 + i$ and $w = -3 + 4i$. Evaluate the following:

(a) (1 mark) $z + w$

(b) (1 mark) zw

(c) (2 marks) $\dfrac{z}{w}$

(d) (3 marks) z^{10}

11. (3 marks) Compute

$$\begin{vmatrix} 3 & -12 & 0 & 0 & 0 \\ 3 & 2 & -1 & -42 & -5 \\ -1 & 0 & 0 & 0 & 0 \\ 2 & -4 & 0 & 3 & -4 \\ 101 & 5 & 0 & 1 & 2 \end{vmatrix}$$

12. (3 marks) Let A, B, and C be $n \times n$ matrices, and let $A^3 = 0$, and $ABC = B$. Prove that $B = 0$.