Math 1823: Final Exam

Name:

Student Number:

1. Evaluate the following limits. If the limit is infinite, state whether it is positive or negative infinity.

   (a) (3 points) $\lim_{x \to 2} \frac{x^2 + 4x + 12}{x + 6}$

   (b) (3 points) $\lim_{x \to 3^+} \frac{x^2 + 9}{x^2 - 9}$

   (c) (3 points) $\lim_{x \to 6} \frac{x^2 - 4x - 12}{x - 6}$

   (d) (3 points) $\lim_{x \to \infty} 3^{-x+2}$

2. (a) (4 points) Let $f(x) = \frac{1}{x+1}$. Prove $f'(x) = -\frac{1}{(x+1)^2}$ using the limit definition of the derivative.

   (b) (2 points) Find the equation of the tangent line to the above function at $x = -2$.

3. (3 points) For what value(s) of k is the following function continuous everywhere?

$$g(t) = \begin{cases} k^2t - 7 & t \leq 3 \\ 2kt^2 + 14 & t > 3 \end{cases}$$

4. Find the derivative for each of the following functions. **DO NOT SIMPLIFY**

(a) (3 points) $y = 2x^3 - \frac{3}{x} + \frac{\sqrt{x}}{2}$

(b) (3 points) $f(x) = (x^2 - 3x + 2)^3(5x - 1)^2$

(c) (3 points) $h(t) = \dfrac{e^t - 1}{e^t + 1}$

(d) (3 points) $y = \ln(x^2 + 1)$

(e) (3 points) $g(u) = \log_4(u^4) + (\log_4 u)^4$

5. (5 points) A farmer wishes to fence in 60,000 m^2 of land in a rectangular field along a straight road. The fencing he plans to use along the road cost \$10/m and the fencing along the other three sides costs \$5/m. Find the dimensions of the field which minimize the cost of the fence.

6. Suppose that the demand function for a certain product is $q = 100 - \frac{1}{12}p^2$.

   (a) (2 points) Find the elasticity of demand.

   (b) (3 points) Find the price which maximizes the revenue.

7. Compute the following integrals:

(a) (3 points) $\int (2x^2 - 1)dx$

(b) (3 points) $\int \left(\frac{2}{x} - 2^x\right) dx$

(c) (3 points) $\int \frac{x}{\sqrt{x^2+1}}dx$

(d) (3 points) $\int_1^2 (x^3 - x)dx$

8. (8 points) Sketch the region which lies between the curves $y = 8 - x^2$ and $y = x^2$ and find its area.

9. (4 points) Find all values x where the tangent line is horizontal:

$$f(x) = \frac{x}{x^2 + 1}$$