University of New Brunswick, Saint John
Department of Mathematical Sciences
Fall 2005

Math 1853
Mathematics for Business I

Final Exam

For credit, circle your instructor's name:
M. Hamdan T. Jones G. Stoica

Name: _____

Student Number: _____

Time: 3 hours

Mark: _____/100

Instructions:

- Show all your work in this booklet.
- Work neatly and in an organized manner.
- If you run out of space in a problem, use the space on the back of the page and clearly indicate where the solution continues.
- Good luck!

$$S = R\left(\frac{(1+i)^n - 1}{i}\right)$$

$$r_{eff} = (1+i)^m - 1$$

1. (10 marks) At age 22, Eric starts a savings account, depositing $1000 at the end of each year. Eric makes no more deposits after age 30. Sarah starts her saving account at age 30 and deposits $1000 at the end of each year. If both accounts pay 10% interest compounded annually, who has more when they retire at age 65? Justify your answer.

2. (8 marks) In 2000, the Smiths bought a $200,000 house, paying a 5% downpayment and secured a 25-year mortgage for the remainder. The interest rate was 6% per year compounded monthly. What is their monthly payment?

3. Differentiate the following:

(a) (4 marks) $f(x) = x^2 \ln x$

(b) (4 marks) $g(x) = \sqrt{3x^3 - 5x + 1}$

(c) (4 marks) $y = \dfrac{x^2 - 4}{(x^2 + 4)^{10}}$

(d) (4 marks) $f(x) = \ln\left(\dfrac{x^2 e^{2x}(3x^2 - 5)}{2x^4(x^2 + 1)}\right)$

4. Let $f(x) = \dfrac{1}{x+1}$

(a) (6 marks) Use the definition of the derivative to find $f'(x)$.

(b) (4 marks) Find the equation of the tangent line to the curve $y = f(x)$ at the point where $x = 1$.

5. Evaluate the following limits, if they exist.

(a) (3 marks) $\lim_{x \to 9} \sqrt{x}(x^2 - 9)$

(b) (3 marks) $\lim_{t \to -2} \dfrac{t^2 + 4}{t + 2}$

(c) (3 marks) $\lim_{r \to 3} \dfrac{3r^2 - 10r + 3}{r^2 - 9}$

(d) (3 marks) $\lim_{x \to \infty} \dfrac{3x^2 - 5x^3 + 6}{2x^3 + x^2 + 1}$

6. The university publisher prints lecture notes for Math 1853. The notes cost \$12 to print, and the publisher must pay the staff \$3000, regardless of the number of lecture notes printed. The lecture notes are then sold for \$20.

   (a) (6 marks) Find the formulas for cost, revenue, and profit

   (b) (1 mark) Calculate the break-even point (in whole numbers of notes)

   (c) (3 marks) Plot the cost and revenue functions, and plot the break-even point.

7. The total cost of producing x refrigerators is given by $C(x) = 30,000 + 700x - 0.01x^2$ for $0 \leq x \leq 1000$.

(a) (2 marks) Find the marginal cost at the production level of 150 refrigerators.

(b) (3 marks) Find the actual cost of producing the 151^{st} refrigerator.

(c) (2 marks) Find the average cost function.

(d) (2 marks) Find the marginal average cost function.

8. Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ -1 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 0 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

(a) (3 marks) Find A^{-1}.

(b) (7 marks) Find $3A - 2CB + 4I$.

9. (5 marks) Solve the following system of equations.

$$\begin{aligned} x - y - z &= 2 \\ 3x - 3y + 2z &= 16 \\ 2x - y + z &= 9 \end{aligned}$$

10. (a) (7 marks) Find the inverse of $R = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$.

(b) (3 marks) Use the inverse found in part (a) to solve the following system of equations.

$$\begin{aligned} 2x + y + z &= 13 \\ 3x + 2y + z &= 4 \\ 2x + y + 2z &= 2 \end{aligned}$$