UNIVERSITY OF NEW BRUNSWICK, SAINT JOHN
DEPARTMENT OF MATHEMATICAL SCIENCES
WINTER 2006

MATH 1853
MATHEMATICS FOR BUSINESS I

Final Exam

Name: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Student Number: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Time: 3 hours

Mark: ⎯⎯⎯⎯⎯/100

Instructions:

- Show all your work in this booklet.

- Do not remove any pages from this booklet.

- You are allowed to use a non-programmable, non-graphing calculator.

- Work neatly and in an organized manner.

- If you run out of space in a problem, use the space on the back of the page and clearly indicate where the solution continues.

- Good luck!

$$S = R\left(\frac{(1+i)^n - 1}{i}\right)$$

$$P = R\left(\frac{1 - (1+i)^{-n}}{i}\right)$$

$$r_{eff} = (1+i)^m - 1$$

1. (10 marks) At age 22, Eric starts a savings account, depositing $1000 at the end of each year. Eric makes no more deposits after age 30. Sarah starts her saving account at age 30 and deposits $1000 at the end of each year. If both accounts pay 6% interest compounded annually, who has more when they retire at age 65? Justify your answer.

2. (8 marks) In 2000, the Smiths bought a $200,000 house and secured a 30-year mortgage for the remainder. The interest rate was 6% per year compounded monthly. What is their monthly payment?

3. Differentiate the following:

(a) (4 marks) $f(x) = xe^{x^2}$

(b) (4 marks) $g(x) = \sqrt{x^3 + 7x^2 - 3}$

(c) (4 marks) $y = \dfrac{x^2 + 1}{(x^2 - 1)^{10}}$

(d) (4 marks) $f(x) = \ln\left(\dfrac{x^2(3x^2 - 5)}{x^2 + 1}\right)$

4. Let $f(x) = \dfrac{1}{x}$

    (a) (6 marks) Use the definition of the derivative to find $f'(x)$.

    (b) (4 marks) Find the equation of the tangent line to the curve $y = f(x)$ at the point $(1, 1)$.

5. Evaluate the following limits, if they exist.

   (a) (3 marks) $\lim\limits_{x \to 4} \sqrt{x}(x^2 - 4)$

   (b) (3 marks) $\lim\limits_{t \to -2} \dfrac{t^2 + 4}{t + 2}$

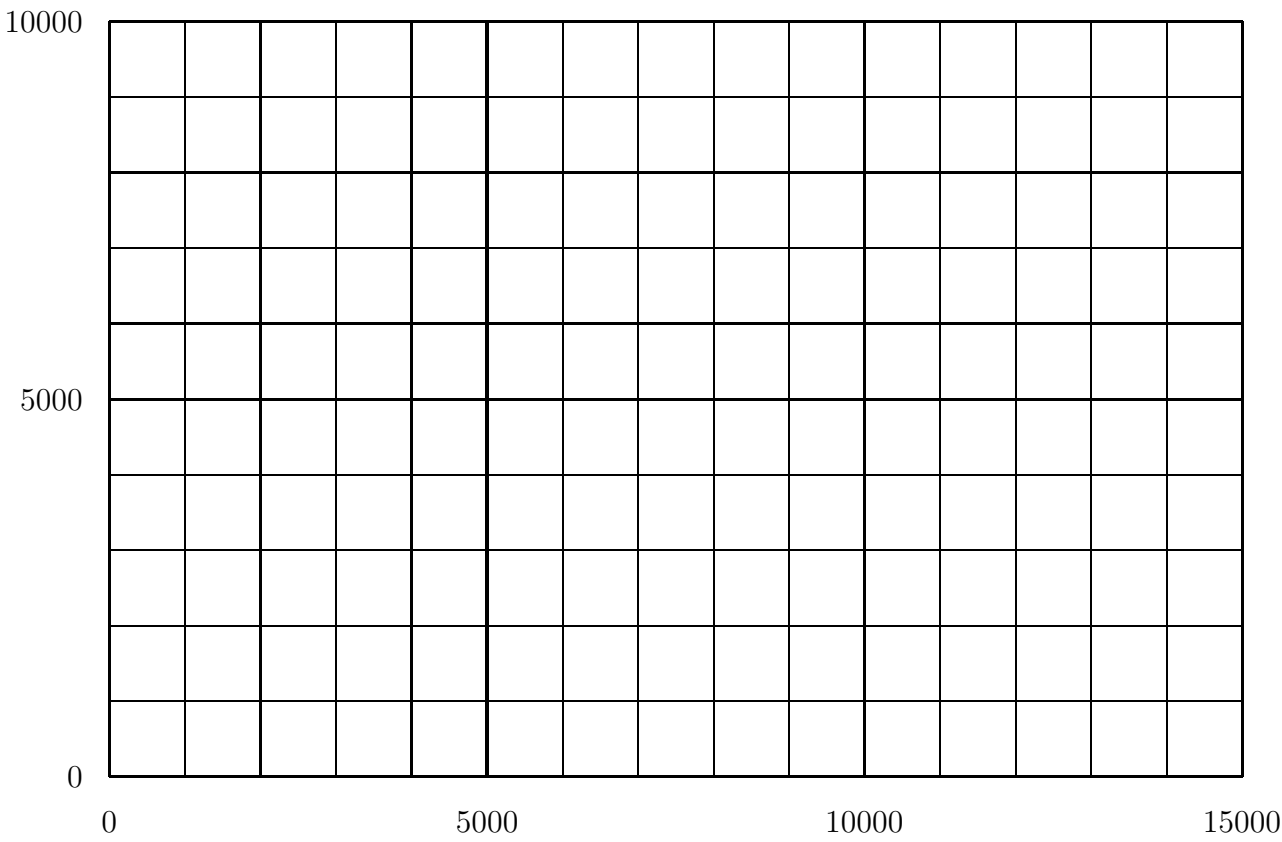   (c) (3 marks) $\lim\limits_{r \to 3} \dfrac{3r^2 - 10r + 3}{r^2 - 9}$

   (d) (3 marks) $\lim\limits_{x \to -1} \dfrac{3x^2 - 5x - 8}{x^2 + 3x + 2}$

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6. A company manufactures bicycle pumps. Each pump sells for $9 and the cost per unit to produce is $3.60, with a fixed cost of $54,000.

   (a) (6 marks) Find the formulas for cost, revenue, and profit

   (b) (1 mark) Calculate the break-even point.

   (c) (3 marks) Plot the cost and revenue functions, and plot the break-even point.

7. The total cost of producing x refrigerators is given by $C(x) = 30,000 + 700x - 0.01x^2$ for $0 \le x \le 1000$.

(a) (2 marks) Find the marginal cost at the production level of 150 refrigerators.

(b) (3 marks) Find the actual cost of producing the 151st refrigerator.

(c) (2 marks) Find the average cost function.

(d) (2 marks) Find the marginal average cost function.

8. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ -1 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 0 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

   (a) (3 marks) Find A^{-1}.

   (b) (7 marks) Find $3A^2 - 2CB + 2I$.

9. (5 marks) Solve the following system of equations.

$$\begin{aligned} x + 2y + z &= -2 \\ -2x - 3y - z &= 1 \\ 2x + 4y + 2z &= -4 \end{aligned}$$

10. (a) (7 marks) Find the inverse of $R = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 3 & 5 \end{bmatrix}$.

(b) (3 marks) Use the inverse found in part (a) to solve the following system of equations.

$$\begin{aligned} x + 2y + 3z &= 13 \\ 2x + y + 2z &= 4 \\ 3x + 3y + 5z &= 2 \end{aligned}$$