MATH 2013, WINTER 2010

FINAL EXAM **APRIL 20, 2009**

Note: You must show your work in order to receive full marks.
No electronic devices are allowed.

1. (10 points) Show that $\overrightarrow{F}(x, y, z) = \langle 2xy+2xz+y^2-z^2, 2xy-2yz+x^2-z^2, -2xz-2yz+x^2-y^2\rangle$ is conservative and find a function ϕ such that $\overrightarrow{F} = \nabla\phi$. Use ϕ to evaluate $\int_C \overrightarrow{F} \cdot \mathrm{d}\overrightarrow{r}$ where C is the arc of a helix going from $(1, 1, 0)$ to $(-1, -1, \pi)$.

2. (10 points) Use Green's Theorem to find $\oint_C xy^2 \, \mathrm{d}x - yx^2 \mathrm{d}y$ where C is the perimeter of the triangle with vertices $(1, 1), (4, 1)$ and $(4, 3)$, travelled in a clockwise direction.

3. (10 points) Evaluate: $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F} = \frac{x}{x^2+y^2}\vec{i} + \frac{y}{x^2+y^2}\vec{j} + \frac{z}{x^2+y^2}\vec{k}$ and S is the surface given by $\vec{r}(u, v) = \langle u\cos v, u\sin v, v\rangle$, for $1 \le u \le 2$ and $0 \le v \le \pi$, with an upward orientation.

4. For this question, let $\vec{F}$ be a vector field whose components have continuous partial derivatives of all orders in an open region (with no holes) that contains the smooth closed surface S which is given a positive (outward) orientation.
   (a) (6 points) Would you use Stoke's Theorem or the Divergence Theorem to simplify $\iint_S \text{curl } \vec{F} \cdot d\vec{S}$? Explain.
   (b) (4 points) Using your choice above, evaluate $\iint_S \text{curl } \vec{F} \cdot d\vec{S}$.

5. (10 points) Find the solution of the following differential equations using the method of your choice: $y'' - 4y' + 4y = e^{2x}$ with initial conditions $y(0) = 1$, $y'(0) = 0$.

6. (10 points) Use power series to solve: $y'' + 2xy' + y = 0$.

7. (15 points) Determine, with justification, the convergence or divergence of the following series:
   (a) $\sum_{n=1}^{\infty} n^{-1-n}$
   (b) $\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$
   (c) $\sum_{n=1}^{\infty} \frac{n2^{n-1}}{3^{n+1}}$

8. (5 points) Determine, with justification, whether the following series is absolutely convergent, conditionally convergent or divergent: $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$

9. (10 points) Find the Taylor series for $f(x) = \frac{1}{\sqrt{2x}}$ centered at $x = 2$. Find the radius of convergence.

10. (10 points) Using the "ϵ-N" definition of convergence, prove that if $\{a_n\}_{n=1}^{\infty}$ converges to 0 and $\{b_n\}_{n=1}^{\infty}$ is bounded, then $\{a_n b_n\}_{n=1}^{\infty}$ converges to 0.