MATH 2213: STUDENT'S NAME:__________________________________ **ID #:**______________

Note: You must show your work in order to receive full marks. Do not detach any pages from this booklet. Use the back of the pages for extra space. No electronic devices allowed.

EXAM **JUNE 19, 2008**

MARKS

Part A: Answer exactly four (4) of the following:

(10) A-1) Show that the distance from the point $P(x_0, y_0, z_0)$ to the plane $ax + by + cz = d$ is

$$\frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}.$$

(10) A-2) Let $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 5 & 2 & 0 \\ 1 & 2 & -1 & 3 \\ -4 & 1 & 0 & 6 \end{bmatrix}$. Find $\det(A)$.

(4) A-3) a) Identify the graph $q = 5x^2 - 6xy - 3y^2 = 12$

(6) b) Rewrite the quadratic form $q = 2x^2 + y^2 + 2xy + 2yz$, so that relative to some orthonormal basis the cross terms disappear.

A-4) Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ y+z \\ z-x \\ z-y \end{bmatrix}$ and $S : \mathbb{R}^4 \to \mathbb{R}^3$ be defined by

$$S\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x+y+z-w \\ x+y-z-w \\ -x-y-z+w \end{bmatrix}.$$

(2) a) Find the matrices of linear transformation for S and T.

(2) b) Find the matrix of linear transformation for ST, and give $ST\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$.

(3) c) Is ST one-to-one? onto? Justify your answer.

(3) d) Find a basis for the kernel of S.

(10) A-5) Find the equation of the line that best fits the following set of points, in the sense of least squares: $\{(-2,-2),(-1,-2),(0,-1),(2,0)\}$.

Part B: Answer exactly two (2) of the following:

(6) B-1) a) Prove: $\overrightarrow{u} \times (\overrightarrow{v} \times \overrightarrow{w}) = (\overrightarrow{u} \cdot \overrightarrow{w})\overrightarrow{v} - (\overrightarrow{u} \cdot \overrightarrow{v})\overrightarrow{w}$.

(4) b) Prove: $\overrightarrow{u} \times (\overrightarrow{v} \times \overrightarrow{w}) + \overrightarrow{v} \times (\overrightarrow{w} \times \overrightarrow{u}) + \overrightarrow{w} \times (\overrightarrow{u} \times \overrightarrow{v}) = \overrightarrow{0}$

(10) B-2) If A is a diagonalizable matrix with all eigenvalues nonnegative, show that A has a square root; in other words, show that $A = B^2$ for some matrix B.

(10) B-3) Let A be a real symmetric matrix. Prove that eigenvectors corresponding to different eigenvalues are orthogonal.