MATH 2213: STUDENT'S NAME:________________________________ ID #:______________

Note: You must show your work in order to receive full marks. Do not detach any pages from this booklet. Use the back of the pages for extra space. No electronic devices allowed.

EXAM **APRIL 19, 2008**

MARKS

Part A: Answer exactly four (4) of the following:

(10) A-1) Find the distance from the line $l : \begin{bmatrix} 1 \\ 6 \\ 1 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ to the plane $x + y - z = 0$.

(10) A-2) Let $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 5 & 2 & 0 \\ 1 & 2 & -1 & 3 \\ -4 & 1 & 0 & 6 \end{bmatrix}$. Find $\det(A)$.

(10) A-3) Rewrite the quadratic form $q = 2x^2 + y^2 + 2xy + 2yz$, so that relative to some orthonormal basis the cross terms disappear. Find the new basis.

(10) A-4) Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ y+z \\ z-x \\ z-y \end{bmatrix}$ and $S : \mathbb{R}^4 \to \mathbb{R}^3$ be defined by $S\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x+y+z-w \\ x+y-z-w \\ -x-y-z+w \end{bmatrix}$.

a) Find the matrices of linear transformation for S and T.

b) Find the matrix of linear transformation for ST, and give $ST\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$.

c) Is ST one-to-one? onto? Justify your answer.

d) Find a basis for the kernel of S.

(10) A-5) Find the equation of the line that best fits the following set of points, in the sense of least squares: $\{(-2,-2),(-1,-2),(0,-1),(2,0)\}$.

Part B: Answer exactly two (2) of the following:

(10) B-1) Prove: Two vectors $\overrightarrow{x}, \overrightarrow{y} \in \mathbb{R}^n$ are orthogonal if and only if $\| \overrightarrow{x} + \overrightarrow{y} \|^2 = \| \overrightarrow{x} - \overrightarrow{y} \|^2$. Also, interpret the result geometrically.

(10) B-2) Let P be a projection matrix. Prove that the column space of P is orthogonal to the null space of P.

(10) B-3) Prove: eigenvectors corresponding to different eigenvalues are pairwise linearly independent.