Math 2513, Fall 2008

FINAL EXAM **DECEMBER 17, 2008**

STUDENT'S NAME:____________________________________ **ID #:**______________

Please circle your instructor's name: Rangipour Jones Seahra

Note: You must show your work in order to receive full marks.
No electronic devices are allowed.

(3) 1. (a) Find the equation of the tangent plane to the surface $x^2 - 2y^2 + z^2 + yz = 2$ at the point $(2, 1, -1)$.

(4) (b) Find the directional derivative of $F(x, y, z) = \sqrt{xyz}$ at the point $(3, 2, 6)$ in the direction $\overrightarrow{v} = \langle -1, -2, 2 \rangle$.

(3) (c) Use the chain rule to find $\dfrac{\partial z}{\partial t}$ if $z = \sin\theta\cos\phi$ with $\theta = st^2$ and $\phi = s^2 t$.

(5) 2. (a) Find and classify all local maxima, local minima, and saddle points of

$$f(x, y) = x^3 + xy^2 + 6x^2 + y^2.$$

(5) (b) Let $f(x, y, z) = 8x - 4z$. Use Lagrange Multipliers to find the maximum and minimum values for f subject to the constraint $x^2 + 10y^2 + z^2 = 5$.

(5) 3. (a) Sketch the region of integration, change the order of integration and evaluate $\int_0^1 \int_{x^2}^1 x\sqrt{1+y^2}\,\mathrm{dydx}$.

(5) (b) Use polar coordinates to evaluate $\iint_R \frac{y}{x}\mathrm{dA}$ where R is the region in the first quadrant which is inside the circle $x^2 + y^2 = 2x$.

(5) 4. (a) Use cylindrical coordinates to evaluate $\iiint_R \sqrt{x^2+y^2}\,\mathrm{dV}$ where R is the region bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane $z = 4$.

(5) (b) Use spherical coordinates to evaluate $\iiint_E z\,\mathrm{dV}$ where E is the solid that lies below the sphere $x^2 + y^2 + z^2 = 1$ and above the cone $z^2 = x^2 + y^2$, in the first octant.

(4) 5. (a) Evaluate $\int_C ((x+yz)\mathrm{dx} + 2x\mathrm{dy} + xyz\mathrm{dz})$ along the arc C given by $x = t,\ y = t^2,\ z = t^3,\ 0 \le t \le 1$.

(6) (b) Show that $\overrightarrow{F}(x,y) = \langle e^x \sin y, e^x \cos y + \sin y\rangle$ is conservative and find a function ϕ such that $\overrightarrow{F} = \nabla\phi$. Use ϕ to evaluate $\int_C \overrightarrow{F} \cdot \mathrm{d}\overrightarrow{r}$ where C is the arc of an ellipse going from $(0,0)$ to $(-1, \frac{\pi}{4})$.

(10) 6. Use Green's Theorem to find $\oint_C (y^3 \mathrm{dx} - x^3 \mathrm{dy})$ where C is the circle $x^2 + y^2 = 4$ travelled counterclockwise.

(8) 7. (a) Given that $\overrightarrow{F} = \langle x^2yz, xy^2z, xyz^2 \rangle$, find the divergence and curl of $\overrightarrow{F}$.

(2) (b) Is it possible to express $\overrightarrow{F}$ of part (a) as the gradient of a function f?

(5) 8. (a) Use Stokes' Theorem to evaluate $\oint_C \overrightarrow{F} \cdot \mathrm{d}\overrightarrow{r}$ where $\overrightarrow{F} = \langle 3, z^2, yz \rangle = 3\,\overrightarrow{i} + z^2\,\overrightarrow{j} + yz\,\overrightarrow{k}$ and C is the boundary of the paraboloid $y = 4 - x^2 - z^2$ in the first quadrant travelled clockwise as viewed from the origin.

(5) (b) Find the curvature of $\overrightarrow{r}(t) = e^t \cos t\,\overrightarrow{i} + e^t \sin t\,\overrightarrow{j} + t\,\overrightarrow{k}$ when $t = 0$.

(10) 9. Use divergence theorem to compute the flux of $\overrightarrow{F} = 2x^3z\,\overrightarrow{i} + 2y^3z\,\overrightarrow{j} + 3z^2\,\overrightarrow{k}$ across the sphere $x^2 + y^2 + z^2 = 4$, oriented outward.