Math 3073 Final Exam, April 15, 2010

(10) 1. Solve for $u(x, y)$:

$$x^2 u_x + y u_y + (x^2 \ln y) u = x^3$$

2. Consider the following PDE: $u_{xx} + 4u_{xy} + 3u_{yy} = 0$, $x, y \in \mathbb{R}$.

(1) (a) Classify as elliptic, parabolic, or hyperbolic.

(3) (b) Write the PDE in standard form. (Hint: complete the square for the differential operator).

(6) (c) Solve the PDE with initial conditions: $u(0, y) = 2y$ and $u_x(0, y) = 6y^2$.

(10) 3. Solve:

$$\begin{aligned} u_t &= 2u_{xx}, & 0 < x < \pi, \quad 0 < t < \infty \\ u_x(0,t) &= u_x(\pi, t) = 0 & \\ u(x, 0) &= \sin x & 0 < x < \pi \end{aligned}$$

(10) 4. Solve $\Delta u = 0$ on the wedge $x^2 + y^2 < a^2$, $0 < y < x$ with the following boundary conditions: $u_\theta = 0$ on $\theta = 0$, $u_\theta = 0$ on $\theta = \pi/4$, and $u_r = \theta$ on $r = a$. (You may assume that all eigenvalues are real and non-negative).

(10) 5. Show that the Green's function for the operator $-\Delta$ on the domain D at the point $x_0 \in D$ is unique.

Helpful formulae:

$$\int \sin x \sin \beta x dx = \frac{\sin((1-\beta)x)}{2(1-\beta)} - \frac{\sin((1+\beta)x)}{2(1+\beta)} + C$$

$$\int \sin x \cos \beta x dx = -\frac{\cos((1-\beta)x)}{2(1-\beta)} - \frac{\cos((1+\beta)x)}{2(1+\beta)} + C$$

$$\int x \sin(\beta x) dx = \frac{\sin(\beta x) - \beta x \cos(\beta x)}{\beta^2} + C$$

$$\int x \cos(\beta x) dx = \frac{\cos(\beta x) + \beta x \sin(\beta x)}{\beta^2} + C$$