University of New Brunswick, Saint John
Department of Mathematical Sciences
Winter 2007

Math 3343
Networks and Graphs

Final Exam

Name: ______________________________

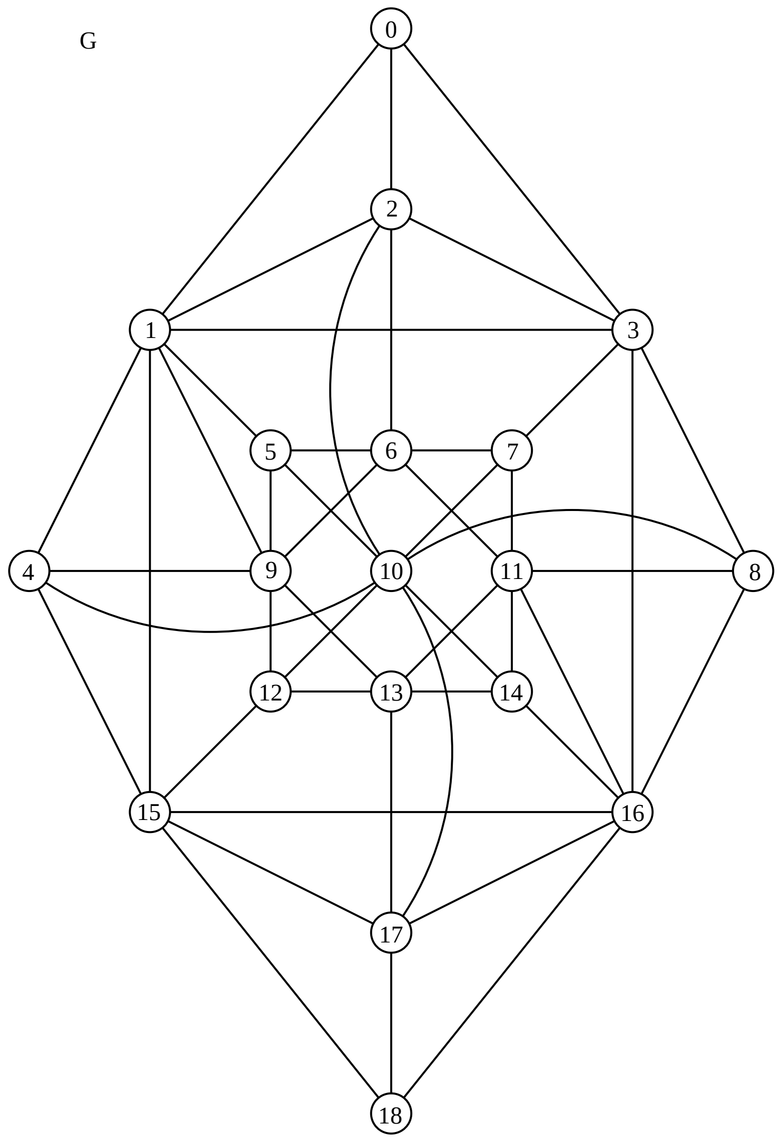
Student Number: ______________________________

Time: 3 hours

Mark: __________/70 (Maximum mark: 76)

Instructions:

- Justification must be given in order to receive full marks.
- Work neatly and in an organized manner.
- Good luck!



1. For the above graph, answer the following questions. Justification must be given in order to get full marks.

(3) (a) Is the graph Eulerian? If so, give the Eulerian circuit.

(5) (b) Is the graph Hamiltonian? If so, give the Hamiltonian cycle.

(3) (c) Is the graph bipartite?

(6) (d) Is the graph planar? If so, redraw without edge crossings.

(3) (e) Write the degree sequence of the graph.

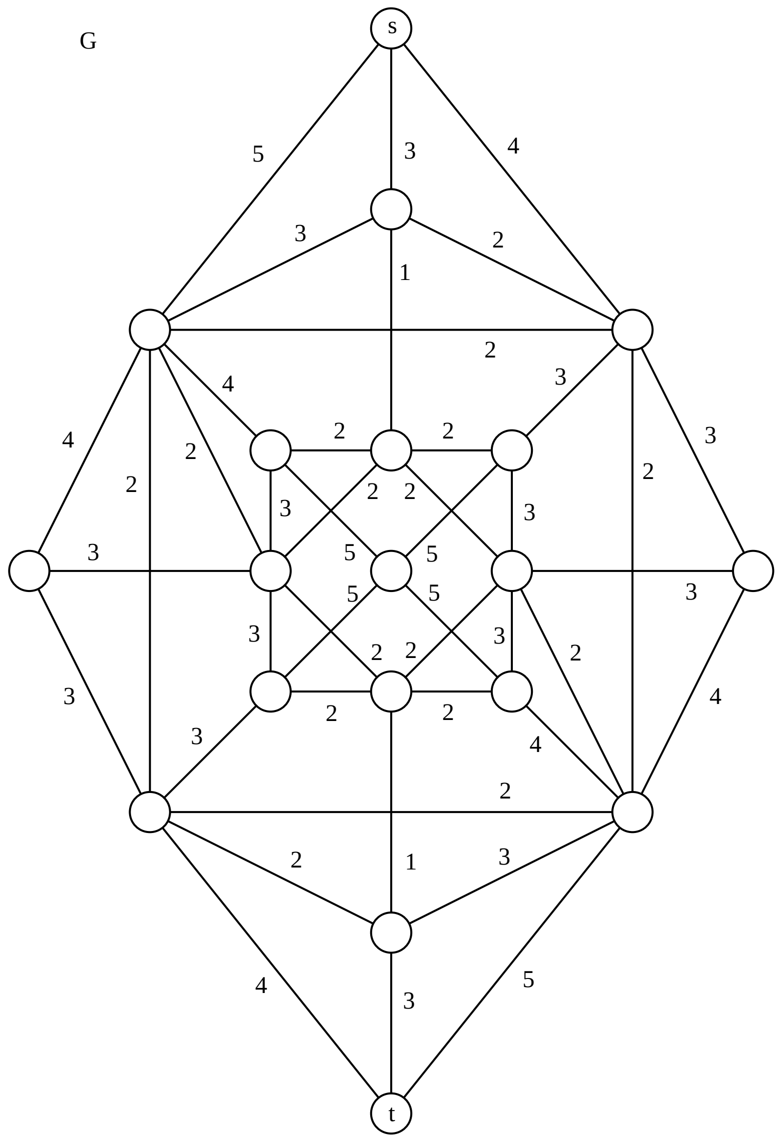
(3) (f) How many edges in the graph?

(5) (g) What is the chromatic number of the graph?

(3) (h) To solve the Chinese Postman problem, which edges need to be added, if any? (You do not need to compute all the possible permutations to answer this question)

(5) (i) Use a depth-first search, starting at 0, to find a spanning tree for this graph. Label the vertices and give the final backtracking.

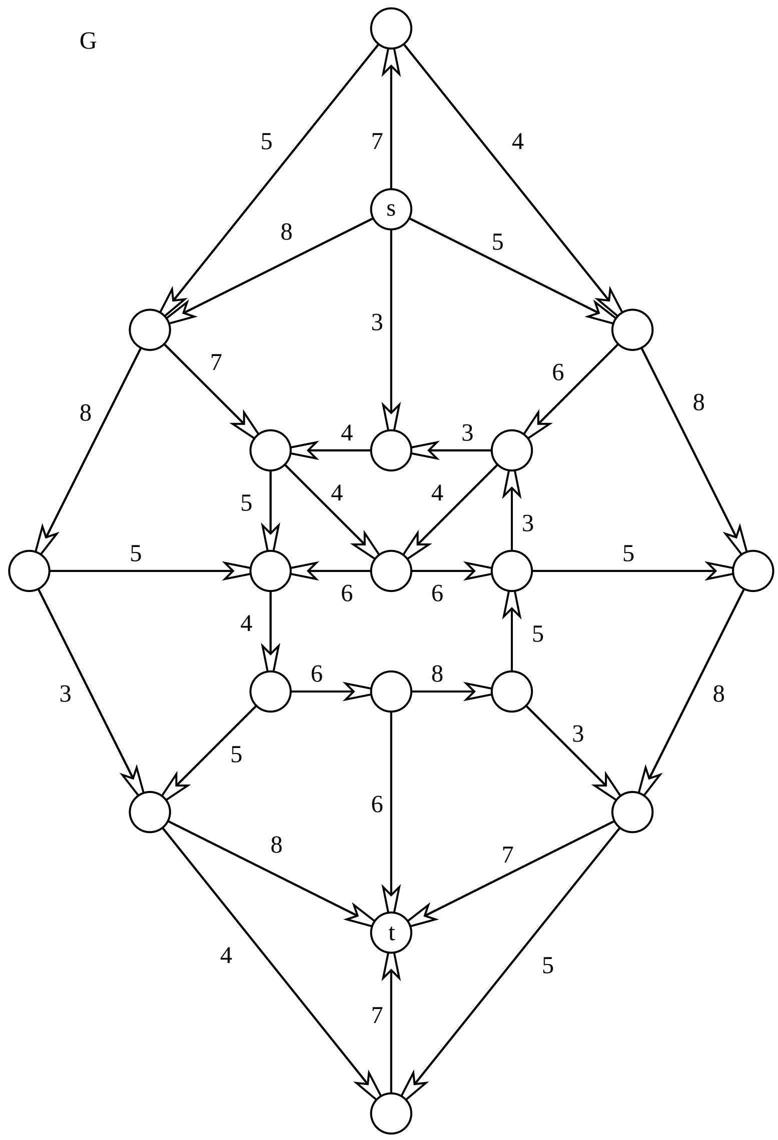
(5) (j) Orient the graph so that it is strongly connected. (Hint: use the result from (1i).)



2. For the above graph, answer the following questions. Justification must be given in order to get full marks.

(6) (a) Find a minimum spanning tree for this graph. Label the edges in the order chosen.

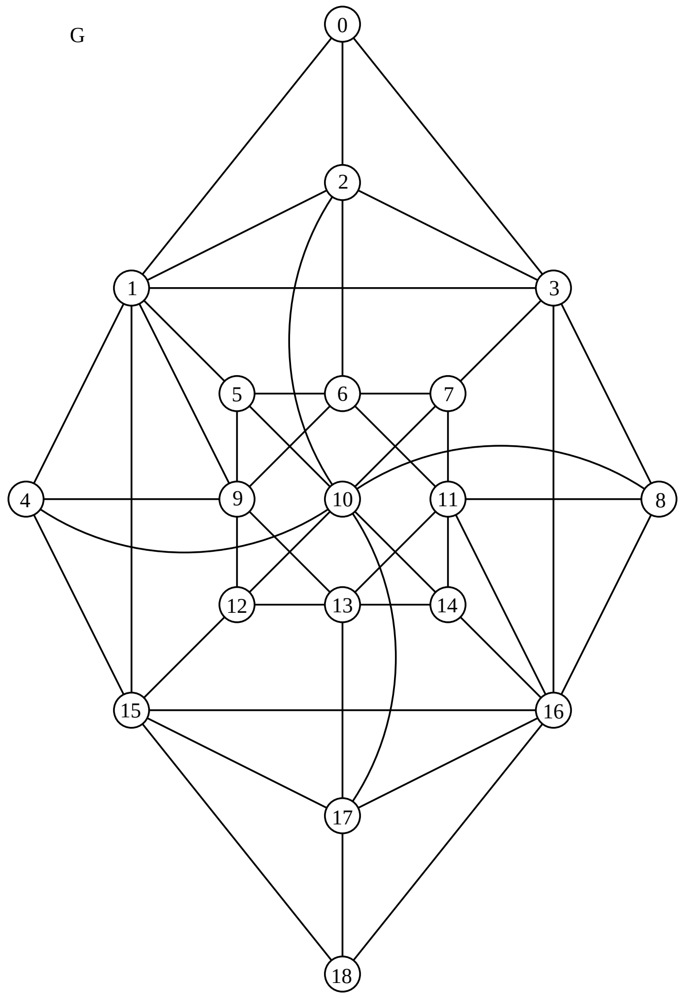
(6) (b) Find the shortest distance from s to t using Dijkstra's original algorithm. Show all the necessary labels, and give the shortest path.



(10) 3. For the above network, find the maximum value of a flow on this network. Give a maximum flow, and the corresponding minimum cut.

(5) 4. (a) Let H be a connected graph with exactly 2 even vertices, and the rest are odd. Prove that there is a subgraph of H which contains all the vertices of H, but omits some edges, with the property that all the vertices in the subgraph are of odd degree. The subgraph need not be connected.

(8) (b) Let H be a connected graph with an even number of even vertices. . Prove that there is a subgraph of H which contains all the vertices of H, with the property that all the vertices in the subgraph are of odd degree.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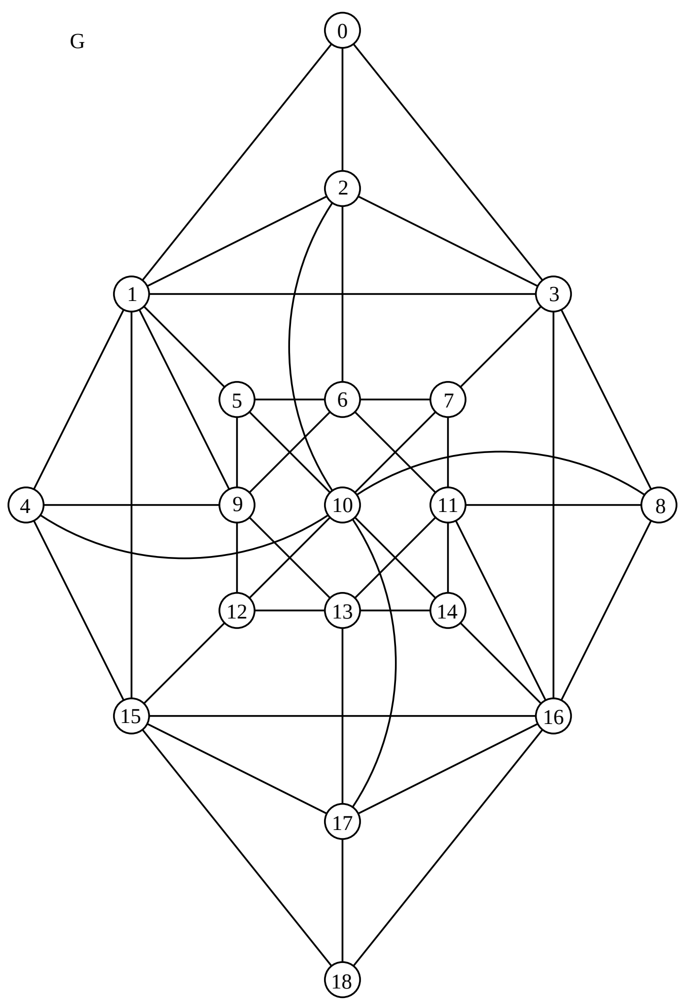
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