Math 317: Complex Analysis
Assignment 1

Due September 19, 2014, by 12:00pm (noon) in Johnson 117A

1. Let z_1, z_2, z_3 be three arbitrary complex numbers. Which of the following equations are true in general? Give reasons. If an equation is false in general, give an example to demonstrate.
   (a) $\overline{z_1 \overline{z_2} z_3} = \overline{z_1} z_2 \overline{z_3}$.
   (b) $\overline{i(z_1 + z_2 + z_3)} = i(\overline{z_1} + \overline{z_2} + \overline{z_3})$.
   (c) $\text{Re}\,(z_1 \overline{z_2} z_3) = \text{Re}\,(\overline{z_1} z_2 \overline{z_3})$.
   (d) $\text{Im}\,(z_1 \overline{z_2} z_3) = \text{Im}\,(\overline{z_1} z_2 \overline{z_3})$.
   (e) $\text{Re}\,(z_1 \overline{z_2} z_3) = \text{Im}\,(i \overline{z_1} z_2 \overline{z_3})$.
2. Sketch the solution to $|z + 1 + i| < 3$.
3. Sketch the set $\{z \mid \text{Re}\, z \leq -1 \text{ or } \text{Im}\, z \geq 0\}$ in the complex plane.
4. Find all the solutions to $z^6 + 64 = 0$. Plot the solutions in the complex plane.
5. Let w be an n^{th} root of unity, i.e., $w^n = 1$, and let $w \neq 1$. Show that
$$1 + w + w^2 + w^3 + \cdots + w^{n-1} = 0$$
6. Let $n \geq 2$ be an integer. Show that the sum of all solutions of $z^n - 1 = 0$ is zero.
7. Show that the roots of a polynomial with real coefficients are either real, or occur in complex conjugate pairs.
8. Prove that $|\text{Re}\, z| + |\text{Im}\, z| \leq \sqrt{2}\,|z|$ for any $z \in \mathbb{C}$.