Math 317: Complex Analysis
Assignment 2

Due October 3, 2014 by 12:00pm (noon) in Johnson 117A

1. If $f(z) = z^3 - 2z^2 + 1$, find $u(x, y) = \text{Re } f(z)$ and $v(x, y) = \text{Im } f(z)$, where $z = x + iy$.
2. Are the following functions analytic?
   (a) $f(z) = x(x^2 - 3 - 3y^3) + iy(3x^2 - y^2 - 1)$
   (b) $g(z) = (x^4 + y^4 - 6x^2y^2) + i4xy(x^2 - y^2)$
3. For each of the following functions of z, indicate the domain in which $f(z)$ is analytic and compute the derivative there.
   (a) $f(z) = \dfrac{z^2 + 2z - 3}{z^3 + 8}$
   (b) $f(z) = (z - 1)^{-10}$
   (c) $f(z) = z\text{Re } z$
4. Show, without using the Cauchy-Riemann equations, that the real and imaginary parts of $f(z) = (z + 1)^4$ are harmonic.
5. Find the harmonic conjugate to $u(x, y) = x + y$ and find the corresponding analytic function $f(z)$. What is $f'(z)$?
6. Let $w = f(z) = u(x, y) + iv(x, y)$. If u, v are continuous at $z = z_0 = x_0 + iy_0$, prove that $f(z)$ is continuous at z_0, using the $\epsilon - \delta$ method.