Math 317: Complex Analysis
Assignment 3

Due October 17, 2014 by 12:00pm (noon) in Johnson 117A

1. If f and g are both analytic functions, prove $\frac{d}{dz}(fg) = \frac{df}{dz}g + f\frac{dg}{dz}$, using the limit definition. You do not need to use the $\epsilon - \delta$ method for limits. (Unrelated side note: the usual quotient rule also holds.)
2. Solve, and sketch the solution in the complex plane.
   (a) Im $(z^2) \leq 2$
   (b) $\left|\frac{z+i}{z-1}\right| = 1$.
3. Find all the solutions of $e^{2iz} = 3$.
4. Find the real and imaginary parts of $f(z) = \cos(z + 1 + i)$. Verify that both are harmonic functions.
5. Find all solutions to $\cos z = 1 + i$.
6. Find, in the form $x + iy$, all values of:
   (a) $(1 + i)^{1+i}$
   (b) $\ln(\sqrt{2} - \sqrt{2}i)$
   (c) the principal value of $(\sqrt{3} + i)^i$
7. Evaluate $\oint_C \left(z + \frac{1}{z}\right) dz$ where C is the unit circle travelled clockwise.
8. Evaluate $\oint_C \text{Re } z dz$ where C is the unit circle travelled counterclockwise.
9. Evaluate $\int_C (z^2 + 3z + 1)dz$ where C is the straight line from $-1 - i$ to $3 + 3i$.
10. Find $\oint_C \frac{3z-1}{z^2-4}dz$ where C is the circle
    (a) $|z - 2| = 2$
    (b) $|z + 2| = 2$ and
    (c) $|z| = 6$,

    all travelled counterclockwise.
11. Find $\int_{-3i}^{3i} \cos 3z dz$.
12. Evaluate $\int_C \frac{z+2}{z^2+1}dz$ where C is
    (a) the path from -3 to 3 along the real axis,
    (b) the upper semi-circle from -3 to 3,
    (c) the lower semi-circle from -3 to 3.