Math 317: Complex Analysis
Assignment 4

Due October 31, 2014 by 12:00pm (noon) in Johnson 117A

1. Use the Cauchy Integral Formula to evaluate the following integrals:

   (a) $\oint_C \frac{e^{z+1}}{z-1} dz$, where C is the circle $|z-1| = 1$ travelled clockwise,

   (b) $\oint_C \frac{\cosh z}{(z-2)(z-1)} dz$, where C is the circle $\left|z - \frac{3}{2}\right| = 1$ travelled clockwise,

   (c) $\oint_C \frac{ze^z}{(z-1)^2} dz$, where C is any simple closed curve surrounding $z = 1$ travelled counterclockwise.

2. C denote the boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$, where C is described in the counterclockwise sense. Evaluate each of the following integrals:

   (a) $\oint_C \frac{\cos z}{z(z^2+8)} dz$

   (b) $\oint_C \frac{z}{2z+1} dz$

   (c) $\oint_C \frac{\tan(z/2)}{(z-x_0)^2} dz \quad (-2 < x_0 < 2)$

   (d) $\oint_C \frac{\cosh z}{z^4} dz$

3. Prove the Cauchy Integral Formula for the case of $n = 2$, that is, prove, for suitable conditions on f and C, that

   $$f''(z_0) = \frac{1}{\pi i} \oint_C \frac{f(z)}{(z-z_0)^3} dz.$$

   Also, state the conditions imposed on f and C. (**Note:** you may assume, if necessary, the case of $n = 1$ and $n = 0$.)

4. Find the Taylor series for $f(z) = \frac{1}{2z-i}$ about

   (a) $z = 0$

   (b) $z = i$

   and state where they are convergent.

5. Find the first 3 non-zero terms in the Taylor series for $f(z) = \frac{1}{(2+3z^2)^2}$ about $z = 2$.