Math 317: Complex Analysis
Assignment 5

Due (Monday) November 17, 2014 by 12:00pm (noon) in Johnson 117A

1. Find **all** the Laurent series for $f(z) = \dfrac{1}{z^2(z-3)}$ with centre $z = 0$ and state where they are convergent.

2. Find **all** the Laurent series for $f(z) = \dfrac{1}{z^2} + 1$ with centre $z = -i$ and state where they are convergent.

3. (a) Find the first 3 non-zero terms in the Laurent series for $f(z) = \dfrac{e^z}{\sin z}$ with centre $z = 0$ valid near $z = 0$.

   (b) Use the answer in (a) to find $\displaystyle\oint_C \frac{e^z}{\sin z} dz$ where C is $|z| = 1$ travelled counterclockwise.

4. Find all singularities of the given functions. Determine if the singularity is an essential singularity or a pole and, if a pole, determine its order. Also, find the residue at each singularity.

   (a) $z^2 e^{1/z}$.

   (b) $\dfrac{3-2z}{z^3+3z^2}$.

   (c) $z^{-5}\cos z$.

   (d) $\dfrac{2}{1-e^z}$.

5. Find $\displaystyle\oint_C \frac{z+1}{z^4-2z^3}$ where C is $|z| = 3$ traveled clockwise.

6. Integrate $\dfrac{3z^2+2z-4}{z^3-4z}$ around the following paths C in the counterclockwise sense:

   (a) $|z| = 1$

   (b) $|z| = 3$

   (c) $|z-4| = 1$

7. Evaluate the following integrals where C is the unit circle traveled counterclockwise:

   (a) $\displaystyle\oint_C \frac{z}{4z^2-1} dz$

   (b) $\displaystyle\oint_C \frac{z+1}{4z^3-z} dz$

   (c) $\displaystyle\oint_C \frac{z+1}{z^4-2z^3} dz$

   (d) $\displaystyle\oint_C \frac{\sinh z}{2z-i} dz$

   (e) $\displaystyle\oint_C \frac{e^{z^2}}{\cos \pi z} dz$